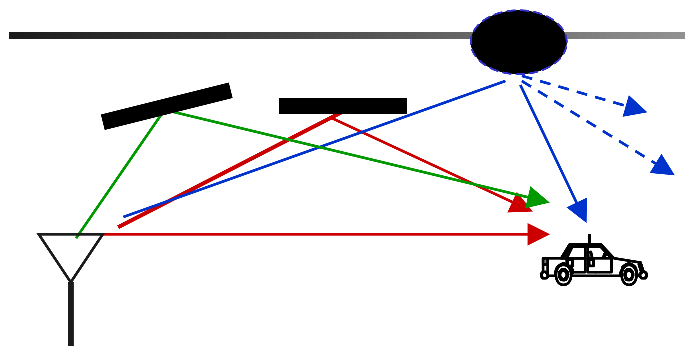
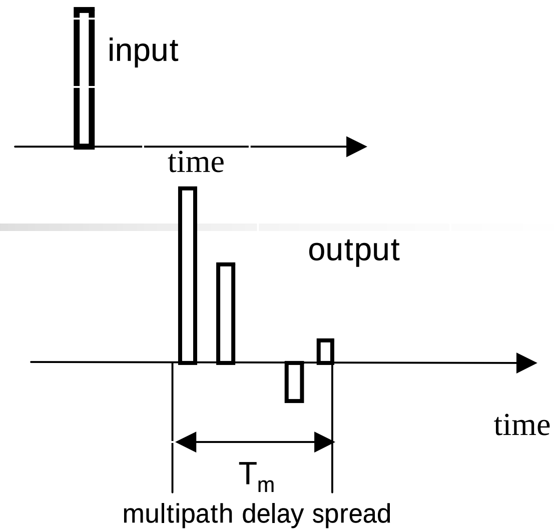
ECSE413B: COMMUNICATIONS SYSTEMS II

Tho Le-Ngoc, Winter 2008

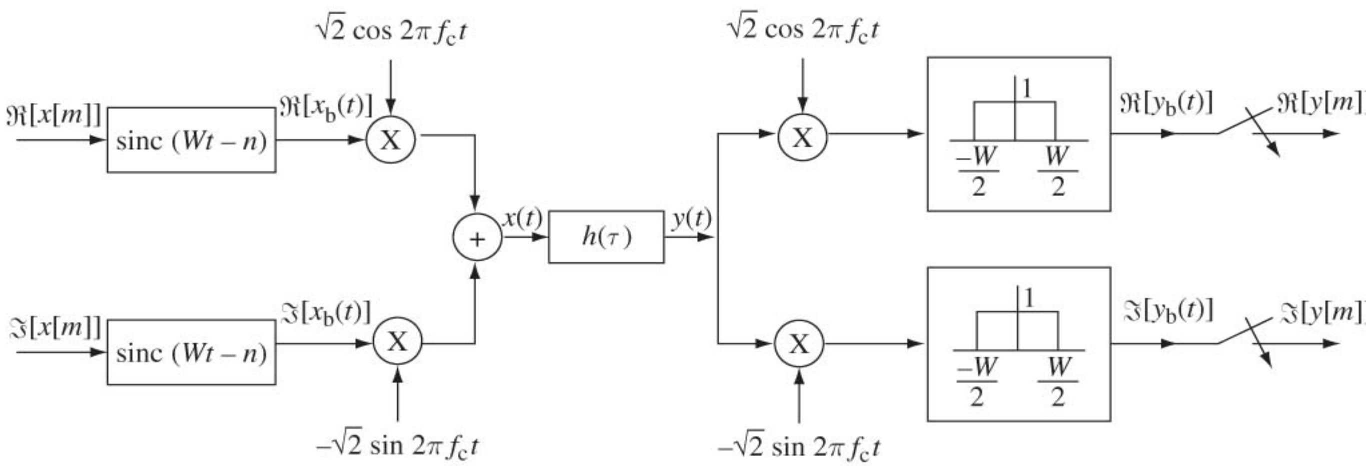
FADING CHANNELS: MODELING

Multipath Modeling





- Channel consists of a random number of path components, each with random amplitude, phase, Doppler shift, delay, changing with time. Multipath fading due to constructive and destructive interference of the transmitted waves.
- W: signal bandwidth, sampling rate: 1/W
- Transmission at passband [f_c-W/2, f_c+W/2] and processing at baseband [-W/2,+W/2].



LINEAR TIME-VARIANT CHANNEL MODEL OF MULTIPATH PROPAGATION

- The (baseband) impulse response of an LTV channel, h(τ; t), is the channel output at t in response to an impulse applied to the channel at (t-τ), i.e., τ is how long ago impulse was put into the channel for the current observation.
- Each path of h(τ; t) is associated with a delay and a complex gain

Multipath channel due to N scatters characterized

by amplitude $\alpha_n(t)$ and delay $\tau_n(t)$ for n=1,2,...,N:

Tx signal: $\mathrm{Re}\left[x(t)e^{-j\omega_c t}\right], x(t): complex-baseband$

$\Rightarrow$ Rx signal (without noise): $\mathrm{Re}\left[r(t)e^{-j\omega_c t}\right]$

$$\text{where } r(t)=\sum_{n=1}^{N}\alpha_n(t)e^{-j\omega_c\tau_n(t)}x(t-\tau_n(t))$$

$$h(\tau;t)=\sum_{n=1}^{N}\alpha_n(t)e^{-j\omega_c\tau_n(t)}\delta(t-\tau_n(t))$$

Linear time-variant (LTV) channel:

Received signal consists of many components with slow amplitude changes, but fast phase changes, introducing constructive and destructive addition of signal components.

$$r(t) = \sum_{n=1}^{N} \alpha_n(t) e^{-j\omega_c \tau_n(t)} x(t - \tau_n(t))$$

$$h(\tau;t) = \sum_{n=1}^{N} \alpha_n(t) e^{-j\omega_c \tau_n(t)} \delta(t - \tau_n(t))$$

$$r(t) = \lim_{\partial\tau \to 0} \sum_{m=-\infty}^{+\infty} h(m\partial\tau;t) x(t - m\partial\tau)\partial\tau = \int_{-\infty}^{+\infty} h(\tau;t) x(t-\tau) d\tau$$

$$\text{input: } x_1(t) \rightarrow \text{ output: } r_1(t) = \int_{-\infty}^{+\infty} h(\tau;t) x_1(t-\tau) d\tau$$

$$\text{input: } x_2(t) = x_1(t - t_1) \rightarrow \text{ output: } r_2(t) = \int_{-\infty}^{+\infty} h(\tau;t) x_1(t - t_1 - \tau) d\tau \neq r_1(t - t_1)$$

$$\text{since } h(\tau;t) \neq h(\tau;t - t_1) \text{ in general}$$

$$\text{Transfer function: } H(f;t) \triangleq \mathbb{F}_\tau \{h(\tau;t)\} = \int_{-\infty}^{+\infty} h(\tau;t) e^{-j2\pi f\tau} d\tau$$

$$H(f;t) \leftrightarrow h(\tau;t) = \mathbb{F}_f^{-1}\{H(f;t)\} = \int_{-\infty}^{+\infty} H(f;t) e^{j2\pi f\tau} df$$

$$R(f;t) = H(f;t) X(f) \text{ where } X(f) = \mathbb{F}_\tau \{x(t)\}$$

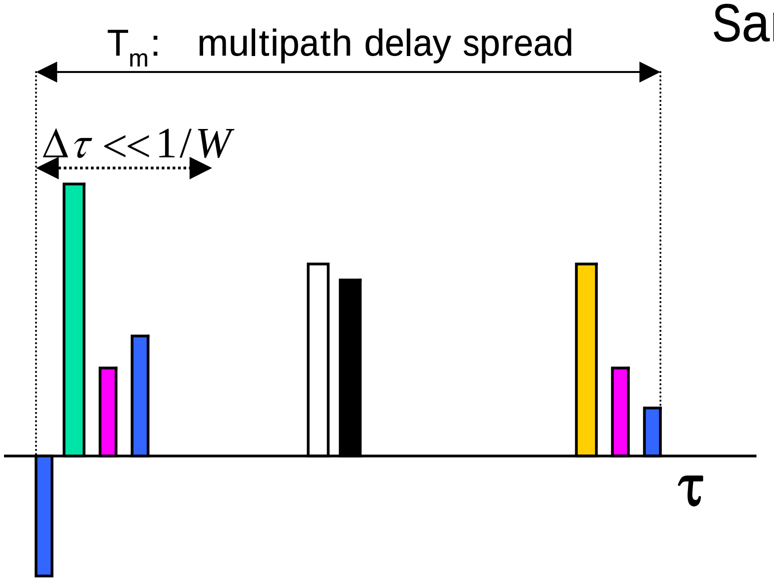
Narrowband frequency-flat fading:

- Delay spread: $T_m = \max_{m,n} |\tau_n(t) - \tau_m(t)|$
- If $T_m \ll 1/W$, W:signal BW, then $x(t) \approx x(t-\tau_n)$.
- Received signal given by

$$r(t) \approx x(t) \sum_{n=1}^{N(t)} \alpha_n(t) e^{-j\varphi_n(t)} \triangleq a(t)x(t)$$

- Multipath effects: complex random fading $a(t)$.
- No signal distortion (spreading in time, frequency-flat fading)
- For $N(t)$ large, the Im and Re parts of a are jointly Gaussian (they are i.i.d., stationary if $\varphi_n(t) \sim U[0,2\pi]$)
- Received signal characterized by its mean, autocorrelation, and cross correlation.

Multipath Resolution and Frequency- Flat and Selective Fading



Sampled baseband-equivalent noise-free received signal:

$$r[m] \triangleq r(m/W) = \sum_{k=K_1}^{K_2} h_m[k]x[m-k] + w[m],$$

$w[m]$: zero-mean complex Gaussian noise

$$h_m[k] \approx \sum_{n^*} \alpha_n(t) e^{-j\omega_c \tau_n(t)}: \text{complex, random tap}$$

- where n^* denotes *all* n's corresponding to delays $\tau_n(t)$'s within the time interval [(k-1)/2W, (k+1)/2W], and
- all possible paths τ_n's are in the time interval [(K_1-1)/2W, (K_2+1)/2W].
- Delay spread $T_m \leq (K_2 - K_1 + 1)/W$. Coherence BW=1/T_m
- Hence, $T_m << 1/W$: single tap (resolvable path), frequency-flat fading
- $T_m > 1/W$: multiple taps (resolvable paths), frequency-selective fading

Time-variant & Doppler effects

$$\tau_n(t) \triangleq \int \tau'_n(t)\,dt \rightarrow \omega_c \tau_n(t) \triangleq 2\pi \int f_c \tau'_n(t)\,dt,$$

$$f_{dn}(t) \triangleq f_c \tau'_n(t): \text{Doppler frequency shift in path } n$$

$$\text{Doppler frequency spread: } B_d = f_c \max_{l,n} \left| \tau'_n(t) - \tau'_l(t) \right|$$

- Coherence time: $(\Delta t)_c = 1/B_d$,
- For $1/B_d$ >>1/W, slow (TIME-FLAT) fading
- For $1/B_d$ < 1/W, fast (TIME-SELECTIVE) fading
- $T_m < 1/B_d$: *under-spread* channels (typical):
  - Delay spread T_m depends on distance to scatterers, of the order of nanoseconds (indoor) to microseconds (outdoor).
  - Coherent time $1/B_d$ depends on carrier frequency and vehicular speed, of the order of milliseconds or more.
  - over a long-time scale, channel can be considered as time-invariant.

STATIONARY RANDOM PROCESS: REVIEW

- **Definition: Strictly** or strict-sense stationary (SSS) random process X(t):
  - Its 1st-order distribution function **independent of t**:
    $F_{X(t)}(x) = F_{X(t+t')}(x)$, for $\forall t,t'$
  - Its 2nd-order distribution function depends only on the **time difference** (between 2 observation times t, t'):
    $F_{X(t),X(t')}(x,x') = F_{X(0),(t'-t)}(x,x')$, $\forall t,t'$
- Results: **Strictly** stationary random process X(t) has:
  - **Constant mean**: $m_X(t) \equiv E\{X(t)\} = m_X$ for $\forall t$
  - **Autocorrelation function** depends only on **time difference**:
    $R_X(t,t') \equiv E\{X(t)X(t')\} = R_X(d) = R_X(-d)$; $R_X(0) = E\{X^2(t)\} \geq |R_X(d)|$, $d = t-t'$
    power spectral density (psd): $$S_X(f) \equiv \int_{-\infty}^{+\infty} R_X(\tau) e^{-j2\pi f\tau} d\tau$$

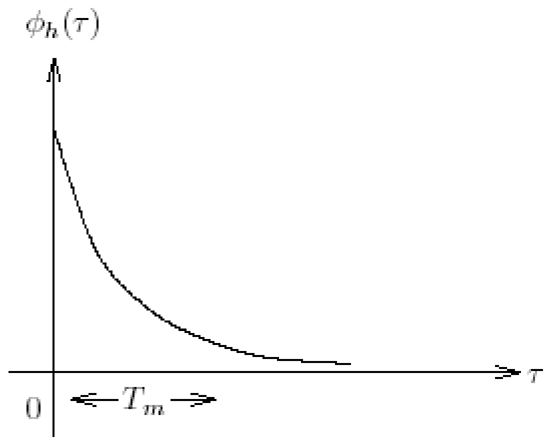
$R_X(t-t')$ describes the interdependence of 2 RV's obtained by observing X(t) at times t and t': $R_X(t-t')$ with a wide pulse-width indicates a slowly fluctuating X(t)

The above results are not sufficient to guarantee that X(t) is **strictly stationary.** If X(t) ONLY has the above characteristics, it is called **wide-sense stationary (WSS)**, or **2nd-order** or **weakly stationary**

WSSUS MODEL OF MULTIPATH CHANNEL

- A channel is wide-sense stationary uncorrelated scattering (WSSUS) when:
  (a) its impulse response $h(\tau; t)$ is a wide-sense stationary (WSS) process;
  (b) US: its impulse responses at τ_1 and τ_2 , $h(\tau_1; t)$ and $h(\tau_2; t)$, are uncorrelated if $\tau_1 \neq \tau_2$ for any t , i.e., $E\{h(\tau_1; t)h(\tau_2; t)\}=0$ if $\tau_1 \neq \tau_2$.
- The **autocorrelation function of $h(\tau; t)$:**
  $\phi_h(\tau,\Delta\tau, \Delta t)\equiv 0.5E\{h^*(\tau,t)\, h(\tau+\Delta\tau,t+\Delta t)\}$; US: $\phi_h(\tau,\Delta\tau, \Delta t) = \phi_h(\tau,0, \Delta t)\delta(\Delta\tau)$
- $\phi_h(\tau) \equiv \phi_h(\tau,0, 0)$: **multipath intensity profile** (or delay power spectrum) provides the average power at the channel output as a function of the propagation delay, τ. Approximate max delay of significant multipath.
- **multipath delay spread**, T_m: nominal width of the multipath intensity profile $\phi_h(\tau)$: range of τ over which $\phi_h(\tau)$ is essentially non-zero
- Usually, it is assumed that $T_m \approx \sigma_\tau$

$$\sigma_\tau = \left[\frac{\int (\tau - \bar{\tau})^2 \phi_h(\tau) d\tau}{\int \phi_h(\tau) d\tau}\right]^{1/2} \qquad \bar{\tau} = \frac{\int \tau \phi_h(\tau) d\tau}{\int \phi_h(\tau) d\tau}$$



autocorrelation function of transfer function H(f; t)

$H(f;t) \triangleq \mathbb{F}_{\tau}\{h(\tau;t)\}$: time-varying channel transfer function

time-frequency correlation function: $\phi_H(\Delta f;\Delta t) \triangleq 0.5E\{H^*(f;t)H(f+\Delta f;t+\Delta t)\}$

$$\phi_H(\Delta f;\Delta t) = \int\int_{-\infty}^{+\infty} 0.5E\{h^*(\tau;t)h(\tau';t+\Delta t)\} e^{-j2\pi(f\tau+\Delta f\tau+\Delta f\Delta\tau)} d\tau d\tau', \quad \tau' = \tau + \Delta\tau$$

WSS: $0.5E\{h^*(\tau;t)h(\tau';t+\Delta t)\} = \phi_h(\tau,\tau',\Delta t)$

US: $E\{h^*(\tau;t)h(\tau';t+\Delta t)\} = E\{h^*(\tau;t)h(\tau;t+\Delta t)\}\delta(\Delta\tau)$

WSSUS: $\phi_h(\tau,\tau',\Delta t) = \phi_h(\tau,0,\Delta t)\delta(\Delta\tau)$;

WSSUS: $\phi_H(\Delta f;\Delta t) = \int_{-\infty}^{+\infty} \phi_h(\tau,0,\Delta t) e^{-j2\pi\Delta f\tau} d\tau = \mathbb{F}_{\tau,\Delta f}\{\phi_h(\tau,0,\Delta t)\}$

frequency-correlation function: for $\Delta t = 0$, $\quad \phi_H(\Delta f) \leftrightarrow \phi_h(\tau) = \mathbb{F}_{\tau,\Delta f}^{-1}\{\Phi(\Delta f)\} = \int_{-\infty}^{+\infty} \phi_H(\Delta f) e^{j2\pi\Delta f\tau} d\Delta f$

US $\Rightarrow \phi_H(\Delta f) = 0.5E\{H^*(f;t)H(f+\Delta f;t)\}$: WSS in frequency

Fourier transform relations for WSSUS h(τ; t)

auto-correlation function of $h(\tau;t)$: $\phi_h(\Delta\tau;\Delta t)=0.5E\{h^*(\tau;t)h(\tau+\Delta\tau;t+\Delta t)\}=\phi_h(\tau,0,\Delta t)\delta(\Delta\tau)$;

for Δt=0, $\phi_h(\tau)$: multipath intensity profile

time-frequency correlation function of $H(f;t)$: $\phi_H(\Delta f;\Delta t)=0.5E\{H^*(f;t)H(f+\Delta f;t+\Delta t)\}$

Fourier transform relations:

$H(f;t)=\mathbb{F}_{\tau,f}\{h(\tau;t)\}$: time-varying channel transfer function

$H(f;v)=\mathbb{F}_{t,vf}\{H(f;t)\}$: Doppler-spread function

$\phi_H(\Delta f;\Delta t)=\mathbb{F}_{\tau,\Delta f}\{\phi_h(\tau;\Delta t)\}$

for Δt=0, $\phi_H(\Delta f)=\mathbb{F}_{\tau,\Delta f}\{\phi_h(\tau)\}$: delay power spectrum,

correlation of channel gains at f and Δf for any t.

$S_H(\Delta f;v)=\mathbb{F}_{\Delta t,v}\{\phi_H(\Delta f;\Delta t)\}$; for Δf=0, Doppler power spectrum $S_H(v)$

$S_H(\Delta f;v)=\mathbb{F}_{\tau,\Delta f}\{S_h(\tau;v)\}$;

$S_H(\Delta f;v)=0.5E\{H^*(f;v)H(f+\Delta f;v+\Delta v)\}$: Auto-correlation function of $H(f;v)$

$S_h(\tau;v)=\mathbb{F}_{\Delta t,v}\{\phi_h(\tau;\Delta t)\}$: scattering function

scattering function:

- measures power vs delay and Doppler
- used to characterize channel rms delay and Doppler spread.

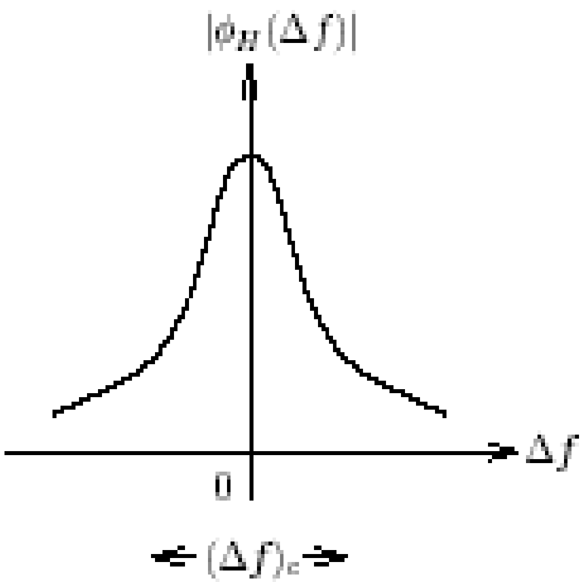
Delay spread & frequency-selective fading

$\phi_H(\Delta f) = \mathbb{F}_{\tau,\Delta f}\{\phi_h(\tau)\}$: delay power spectrum,

correlation of channel gains at f and Δf for any t

$\phi_H(\Delta f)=0$ implies signals separated in frequency by Δf will be uncorrelated after passing through channel

coherence bandwidth of the channel, $(\Delta f)_c \approx 1/T_m$: The maximum frequency difference for which the signals are still strongly correlated. Two sinusoids with frequency separation larger than $(\Delta f)_c$ are affected differently by the channel at any t.



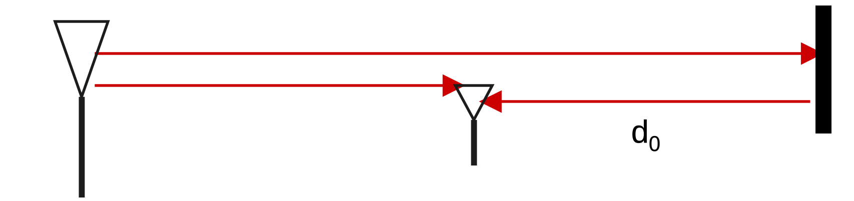
W: bandwidth of the transmitted signal.

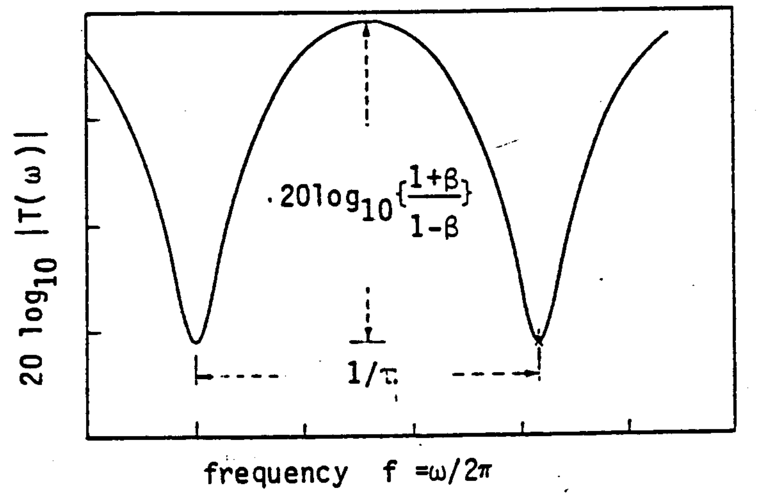
If $(\Delta f)_c$ < W, frequency-selective fading channel: severe ISI;

If $(\Delta f)_c$ >> W, flat fading channel: negligible ISI.

ISI-free channel: $\phi_H(\Delta f) \approx \phi_o$:constant $\leftrightarrow \phi_h(\tau) = \phi_o \delta(\tau)$

EXAMPLE OF 2-PATH MODEL





(a) AMPLITUDE DISTORTION (| T(ω)| in dB)

At receiver, the received signal is

$r(t) = x(t) + \beta\, x(t-\tau)$

where x(t) : the main path

β : relative level between the main and reflected paths

$\tau = 2d_0/c$: relative time delay between the main and reflected path,

Channel transfer function $T(\omega) = 1 + \beta e^{-j\omega\tau}$

Amplitude distortion:

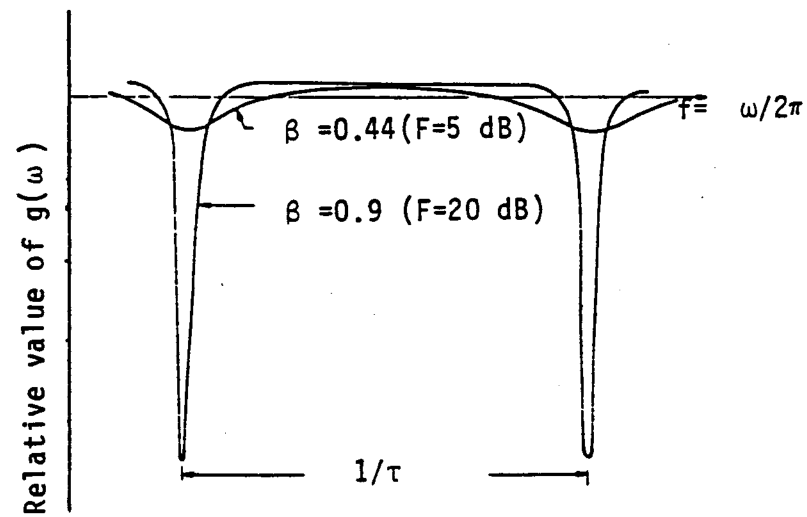
$|T(\omega)|^2 = 1 + \beta^2 + 2\beta\cos\omega\tau = 1 + \beta^2 + 2\beta\cos\omega\tau$

phase distortion:

$\Phi(\omega) = \tan^{-1}[\beta\sin\omega\tau/(1 + \beta\cos\omega\tau)]$

group delay distortion $g(\omega) = d\Phi/d\omega$

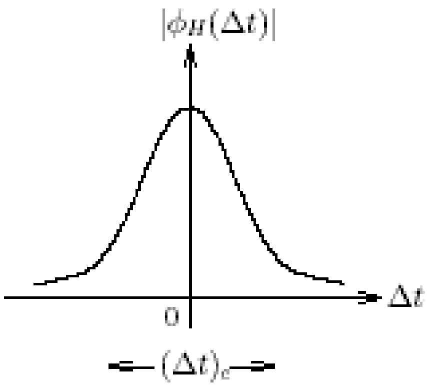
$g(\omega) = \beta\tau(\beta + \cos\omega\tau)/(1 + \beta^2 + 2\beta\cos\omega\tau)$

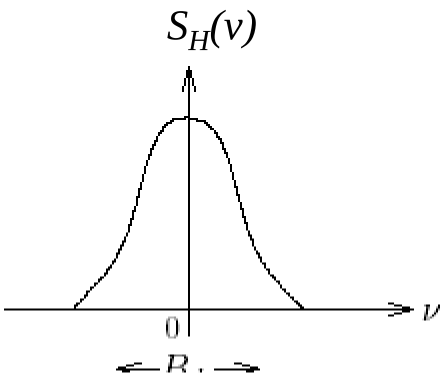


(b) GROUP DELAY DISTORTION (g(ω))

Time correlation function $\phi_H(\Delta t)$ & Doppler power spectrum

- $\phi_H(\Delta t)$ is independent of f due to the US assumption: US in the time domain is equivalent to WSS in the frequency domain.
- $\phi_H(\Delta t)$ characterizes, on average, how fast the channel transfer function changes with time at each frequency.
- $\phi_H(\Delta t)=0$ implies signals separated in time by Δt will be uncorrelated after passing through channel.
- **coherence time** of the fading channel, $(\Delta t)_c$: Maximum time over which $\phi_H(\Delta t)>0$: nominal width of $\phi_H(\Delta t)$.
- $(\Delta t)_c >> T$(symbol interval of the Tx signal): slow fading.
- Doppler power spectrum $\phi_H(\nu)$: Fourier transform of the time correlation function $\phi_H(\Delta t)$
- Doppler spread B_d is maximum Doppler for which $\phi_H(\nu)\to 0$: nominal width of $\phi_H(\nu)$, $B_d \approx 1/(\Delta t)_c$





Time correlation function $\phi_H(\Delta t)$ & Doppler power spectrum

$H(f;t) = \mathbb{F}_{\tau,f}\{h(\tau;t)\}$: time-varying channel transfer function

time-frequency correlation function of $H(f;t)$:

$\phi_H(\Delta f;\Delta t) = 0.5E\{H^*(f;t)H(f+\Delta f;t+\Delta t)\}$

WSSUS $\rightarrow \phi_H(\Delta f;\Delta t) = \mathbb{F}_{\tau,\Delta f}\{\phi_h(\tau;\Delta t)\}$

for Δt=0, $\phi_H(\Delta f) = \mathbb{F}_{\tau,\Delta f}\{\phi_h(\tau)\}$: delay power spectrum,

correlation of channel gains at f and Δf for any t.

$S_H(\Delta f;\nu) = \mathbb{F}_{\Delta t,\nu}\{\phi_H(\Delta f;\Delta t)\}$;

for Δf=0, $\phi_H(\Delta t)$: time-correlation function of $H(f;t)$, Doppler power spectrum $S_H(\nu)$

$\left(S_H(\Delta f;\nu) = \mathbb{F}_{\tau,\Delta f}\{S_h(\tau;\nu)\},\ S_h(\tau;\nu) = \mathbb{F}_{\Delta t,\nu}\{\phi_h(\tau;\Delta t)\}\text{: scattering function}\right)$

$$\bar{\nu} = \left[\int \nu S_H(\nu)d\nu\right]\left[\int S_H(\nu)d\nu\right]^{-1}, \quad \sigma_\nu^2 = \left[\int[\nu-\bar{\nu}]^2 S_H(\nu)d\nu\right]\left[\int S_H(\nu)d\nu\right]^{-1}$$

Doppler spread: $B_d \approx \sigma_\nu$

Doppler-spread function H(f,ν):

$H(f;t)=\mathbb{F}_{\tau,f}\{h(\tau;t)\}$: channel transfer function $\rightarrow H(f;\nu)=\mathbb{F}_{t,\nu f}\{H(f;t)\}$: Doppler-spread function

$\phi_H(\Delta f;\Delta t)=\mathbb{F}_{\tau,\Delta f}\{\phi_h(\tau;\Delta t)\}$: time-frequency correlation function of $H(f;t)$

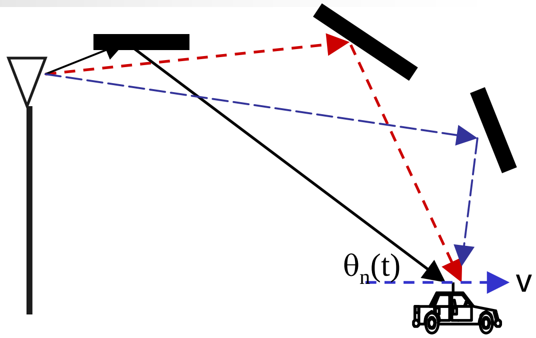
for $\Delta t=0$, $\phi_H(\Delta f)=\mathbb{F}_{\tau,\Delta f}\{\phi_h(\tau)\}$: delay power spectrum, correlation of channel gains at f and Δf for any t.

$S_H(\Delta f;\nu)=\mathbb{F}_{\Delta t,\nu}\{\phi_H(\Delta f;\Delta t)\}$; for $\Delta f=0$, Doppler power spectrum $S_H(\nu)$

$S_H(\Delta f;\nu)=0.5E\{H^*(f;\nu)H(f+\Delta f;\nu+\Delta\nu)\}$: Auto-correlation function of $H(f;\nu)$

- being time-variant in the time domain can be equivalently described by having Doppler shifts in the frequency domain.
  - **DOPPLER-SPREAD ⇒ TIME-SELECTIVE FADING**
- Doppler power spectrum: function of the Doppler shift, ν, Fourier transform of the time-correlation function $\phi_H(\Delta t)$

Doppler Spread in land-mobile channel



$$h(\tau;t)=\sum_{n=1}^{N(t)}\alpha_n(t)e^{-j\omega_c\tau_n(t)}\delta(t-\tau_n(t))$$

$$\omega_c\tau_n(t)=\varphi_{n0}+2\pi\int f_{dn}(t)dt,\quad \varphi_{n0}=2\pi d_n/c,$$

$f_{dn}(t)=f_c\left[v(t)/c\right]\cos\theta_n(t)$: Doppler frequency shift in path n

$\rightarrow$ Doppler frequency spread:

$$B_d=\max_{l,n}\left|f_{dn}(t)-f_{dl}(t)\right|=2f_c\left[v(t)/c\right]$$

$$H(f;t)=\mathbb{F}_{\tau,f}\left\{h(\tau;t)\right\}=\sum_{n=1}^{N(t)}\alpha_n(t)e^{-j2\pi\left[(f/f_c)-1\right]\left[\varphi_{n0}+2\pi\int f_{dn}(t)dt\right]}$$

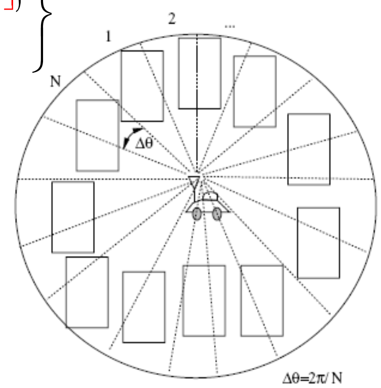
$$\phi_H(\Delta t)=0.5\sum_{l=1}^{N(t)}\sum_{n=1}^{N(t)}E\left\{\alpha_n{}^*(t)\alpha_l(t+\Delta t)e^{j\left[(f/f_c)-1\right]\left\{\left[\varphi_{n0}+2\pi\int f_{dn}(t)dt\right]-\left[\varphi_{l0}+2\pi\int f_{dl}(t+\Delta t)dt'\right]\right\}}\right\}$$

For independent n,l terms and uniformly distributed θ_n:

$$\phi_H(\Delta t)=0.5\sum_{n=1}^{N(t)}E\left\{\left|\alpha_n\right|^2\right\}E_{\theta_n}\left\{e^{-j\pi B_d\Delta t\cos\theta_n}\right\}=PJ_0(\pi B_d\Delta t),$$

$$J_0(x)=\frac{1}{\pi}\int_0^{\pi}e^{-jx\cos\theta}d\theta,\quad 2P=\sum_{n=1}^{N(t)}E\left\{\left|\alpha_n\right|^2\right\}$$

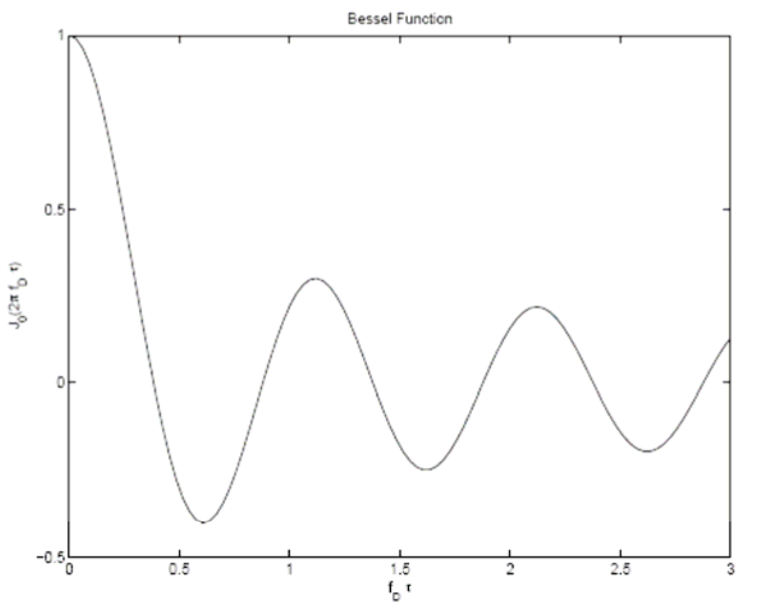
uniform scattering environment
[Clarke, Jakes]

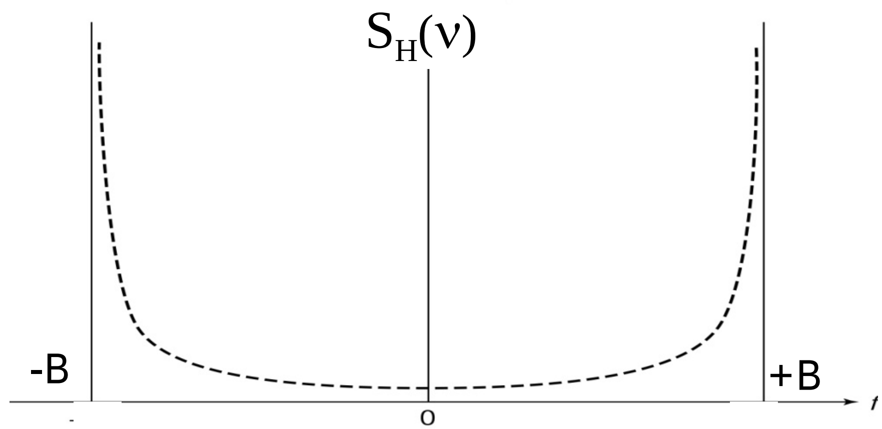


AUTOCORRELATION & DOPPLER POWER SPECTRUM OF A LAND-MOBILE RADIO CHANNEL

- Correlation over Time can be specified by autocorrelation function and power spectral density of fading process.

- For an omnidirectional mobile antenna and received plan waves uniformly distributed in arrival angle,
  - **time correlation function**:
    - $\phi_H(\Delta t) = PJ_o(2\pi B\Delta t)$,
    - $J_o(x)$ is the 0^{th}-order Bessel function of the 1^{st} kind.
  - **Doppler power spectrum** $\phi_H(\nu)$: Fourier transform of time-correlation function,
    - $S_H(\nu)=P/\{\pi[B^2- \nu^2]^{1/2}\}$,
    - $|\nu| < B=B_d/2=vf_c/c$





STATISTICAL MULTI-TAP MODELS

- multi-tap model for design and performance analysis based on statistical ensemble of channels rather than specific physical channel

$$r[m] \triangleq r(m/W) = \sum_{k=K_1}^{K_2} h_m[k] x[m-k] + w[m],$$

$$w[m]: \text{zero-mean complex Gaussian noise}$$

$$h_m[k] \approx \sum_{n^*} \alpha_n(t) e^{-j\omega_c \tau_n(t)}: \text{complex, random tap}$$

- Non-LOS: many small scattered paths, complex circular symmetric Gaussian tap. $\rightarrow$ signal envelope follows Rayleigh distribution (power is exponential)
- Near-LOS (with LOS component): 1 line-of-sight plus scattered paths. $\rightarrow$ signal envelope follows Ricean distribution.
- In some environments, measured results support Nakagami distribution (Similar to Ricean, but models “worse than Rayleigh”, better to obtain closed-form BER expressions)

Small-Scale Multipath Fading:

Rayleigh fading (NLOS propagation) case

$$r(t) = \sum_{n=1}^{N} \alpha_n(t) e^{-j2\pi f_c \tau_n(t)} x(t-\tau_n(t))$$
$$\approx [\sum_{n=1}^{N} \alpha_n(t) e^{-j2\pi f_c \tau_n(t)}] x(t-\bar{\tau}).$$

$$Z(t) = \sum_{n=1}^{N} \alpha_n(t) e^{-j2\pi f_c \tau_n(t)}$$
$$= Z_c(t) - jZ_s(t)$$

$$Z_c(t) = \sum_{n=1}^{N} \alpha_n(t) \cos\theta_n(t)$$

$$Z_s(t) = \sum_{n=1}^{N} \alpha_n(t) \sin\theta_n(t)$$

$$Z(t) = \alpha(t)\exp[j\theta(t)]$$

central limit theorem: when N is sufficiently large, $Z_c(t)$ and $Z_s(t)$ are approximately independent Gaussian random variables with zero mean and equal variance

$$\sigma_z^2 = \frac{1}{2}\sum_{n=1}^{N} E[\alpha_n^2]$$

$$f_{Z_c Z_s}(x,y) = \frac{1}{2\pi\sigma_z^2}\exp[-\frac{x^2+y^2}{2\sigma_z^2}],$$

The amplitude fading, α, follows a Rayleigh distribution with parameter σ_z^2

$$f_\alpha(x) = \begin{cases} \frac{x}{\sigma_z^2}\exp(-\frac{x^2}{2\sigma_z^2}), & x \geq 0 \\ 0, & x < 0 \end{cases}$$

The phase distortion follows the uniform distribution over $[0, 2\pi]$,

$$\alpha(t) = \sqrt{Z_c^2(t) + Z_s^2(t)}, \quad \theta(t) = \tan^{-1}[Z_s(t)/Z_c(t)]$$

Small-Scale Multipath Fading: Rician Fading (LOS propagation) case

$$Z(t) = Z_c(t) - jZ_s(t) + \Gamma(t)$$

$\Gamma(t) = \alpha_0(t)e^{-j\theta_0(t)}$ is the deterministic LOS component

$$f_\alpha(x) = \underbrace{\frac{x}{\sigma_z^2}\exp(-\frac{x^2}{2\sigma_z^2})}_{\text{Rayleigh}} \cdot \underbrace{\exp\{-\frac{\alpha_0^2}{2\sigma_z^2}\} \cdot I_0(\frac{\alpha_0 x}{\sigma_z^2})}_{\text{modifier}}$$

α_0^2: power of the LOS component

$$= \frac{x}{\sigma_z^2}\exp(-\frac{x^2+\alpha_0^2}{2\sigma_z^2})I_0(\frac{\alpha_0 x}{\sigma_z^2}), \quad x \geq 0,$$

$$I_0(x) = \frac{1}{2\pi}\int_0^{2\pi} \exp(x\cos\theta)d\theta.$$

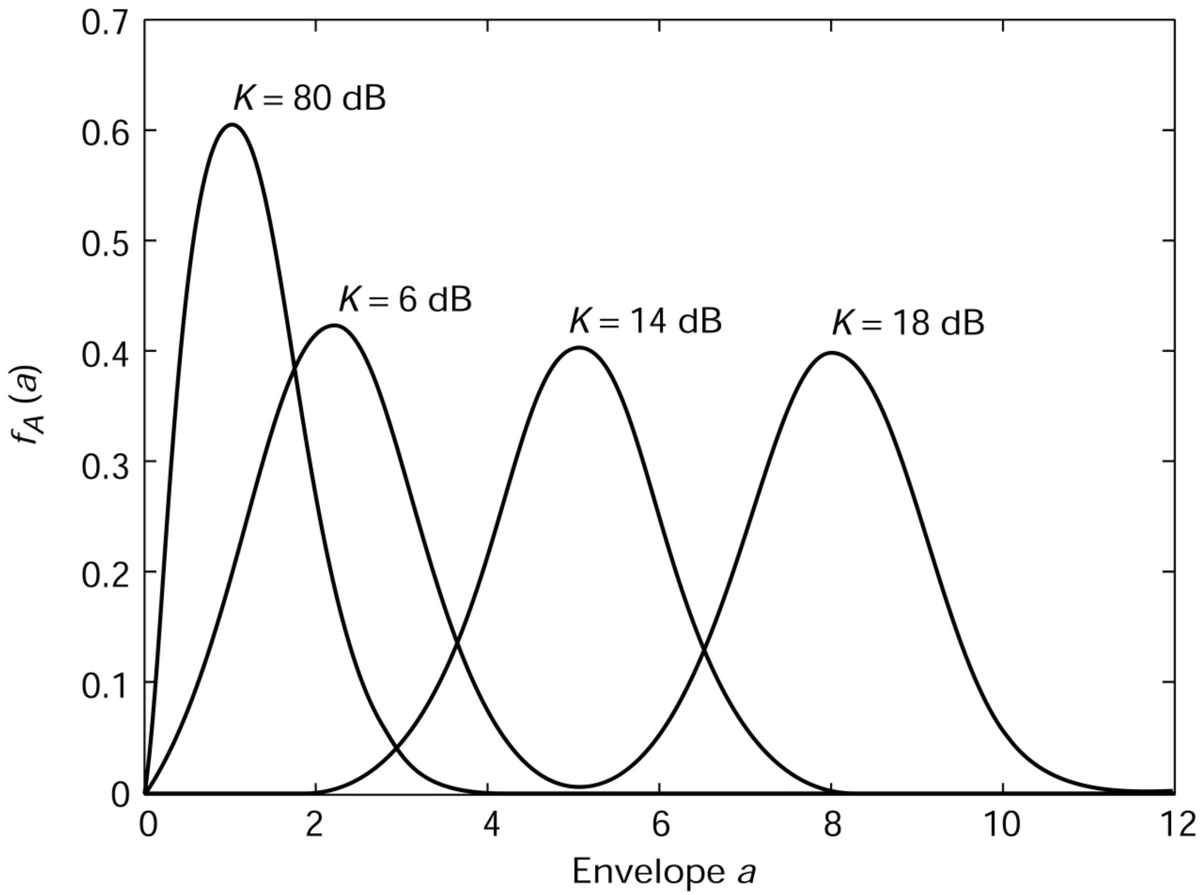
zero-order modified Bessel function of the first kind

$$K \triangleq \frac{\text{Power of the LOS component}}{\text{Total power of all other scatterers}} = \frac{\alpha_0^2}{2\sigma_z^2}.$$

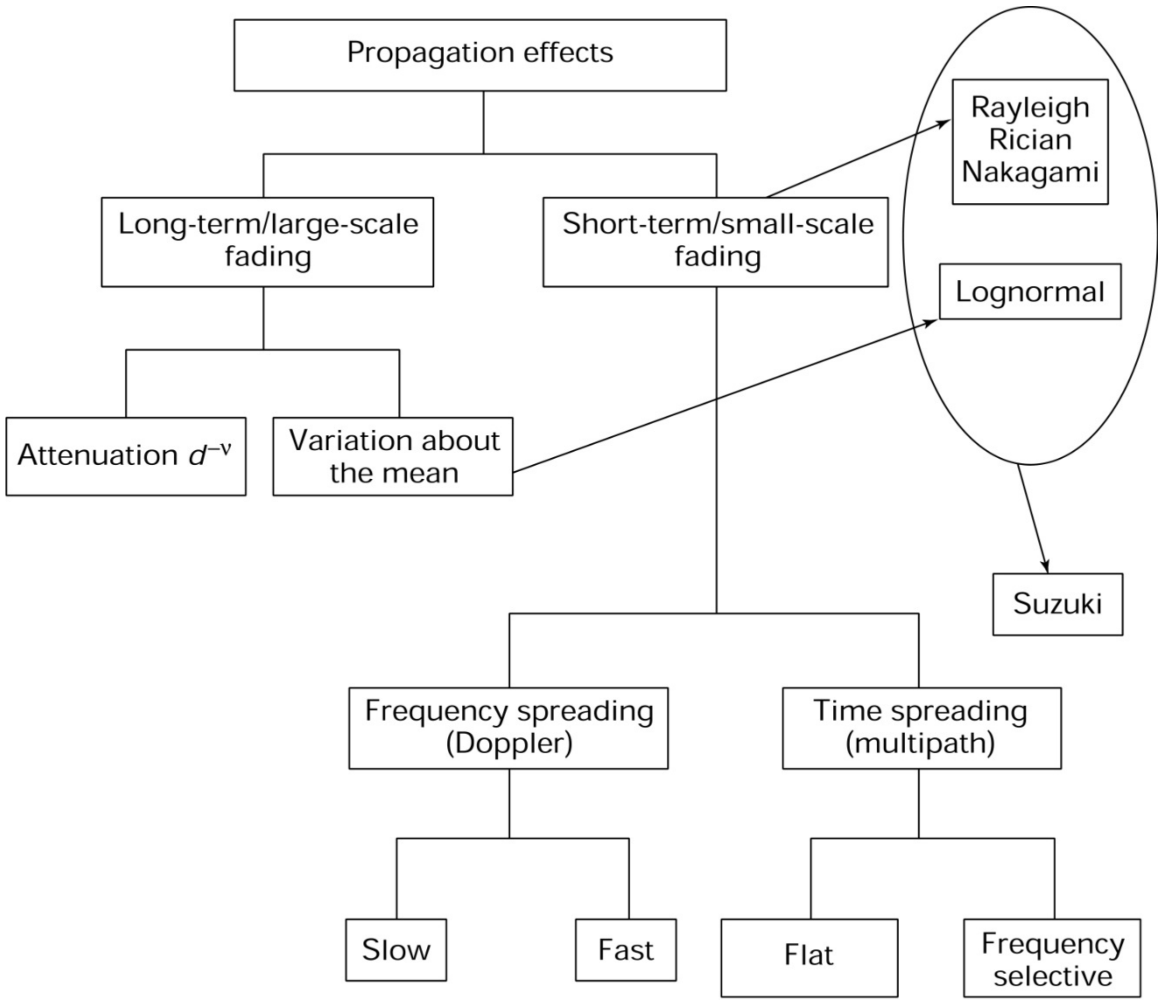
K = 0: Rayleigh

K $\rightarrow\infty$: no fading

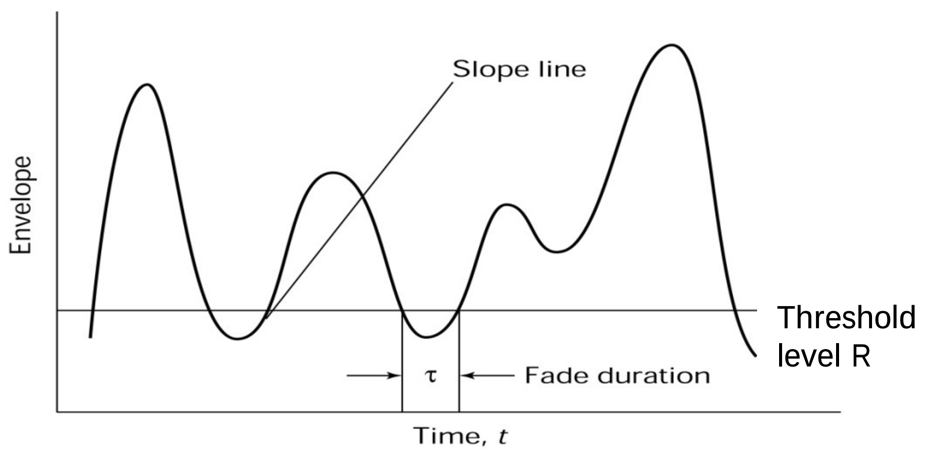
Rician probability density functions with $\sigma_z^2 = 1$



attenuation and fading



LEVEL CROSSINGS & FADE DURATION



fade duration:

- a user is in continuous outage since the actual SNR (γ) is below the threshold level R required to maintain a maximum BER
- can be derived from level crossing rate of fading process
- for Rayleigh fading,
  - Inversely proportional to Doppler frequency
  - Dependent on margin

$z(t) \triangleq |r(t)|$: stationary and ergodic,

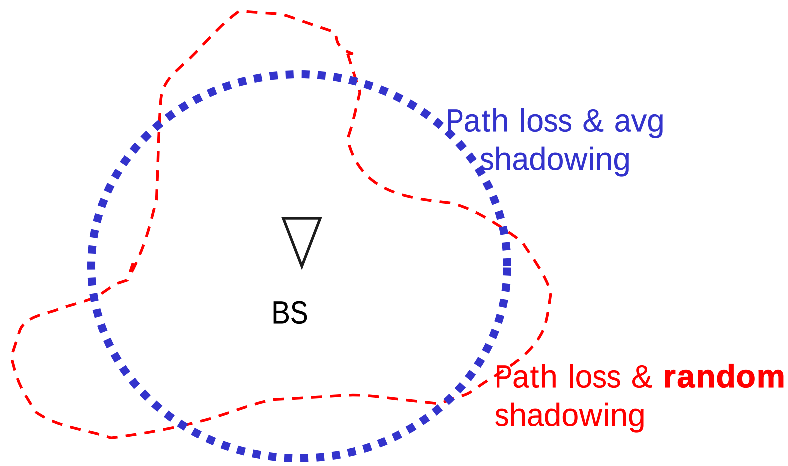
$\bar{z} \triangleq E\{|r(t)|\}$, threshold R, i.e., $z(t) < R$: outage, fade

t_i: fade duration, for large observation time T, $\Pr\{z(t) < R\} = \left[\sum_i t_i\right] / T$

average fade duration: $\bar{t}_R = \dfrac{(e^{(R/\bar{z})^2} - 1)}{(R/\bar{z})\, f_d \sqrt{2\pi}}$

Outage Probability and Cell Coverage Area

- **Outage**: received power below given minimum required for acceptable performance.
- **cell coverage area** : expected percentage of area within a cell that has received power above a given minimum required for acceptable performance.
- circular cells for path loss only,
- amoeba cells for path loss & shadowing as tradeoff between coverage and interference
- Cell coverage area increases as shadowing variance decrease



EQUAL RX POWER CONTOURS

REFERENCES

- J. Mark, W. Zhuang, Wireless Communications and Networking, Prentice-Hall, 2003
- T.S. Rappaport, Wireless Communications: Principles and Practice, 2nd Ed, Prentice-Hall, 2002
- P.M. Shankar, Introduction to Wireless Systems, John Wiley & Sons, 2002
- and materials from various sources