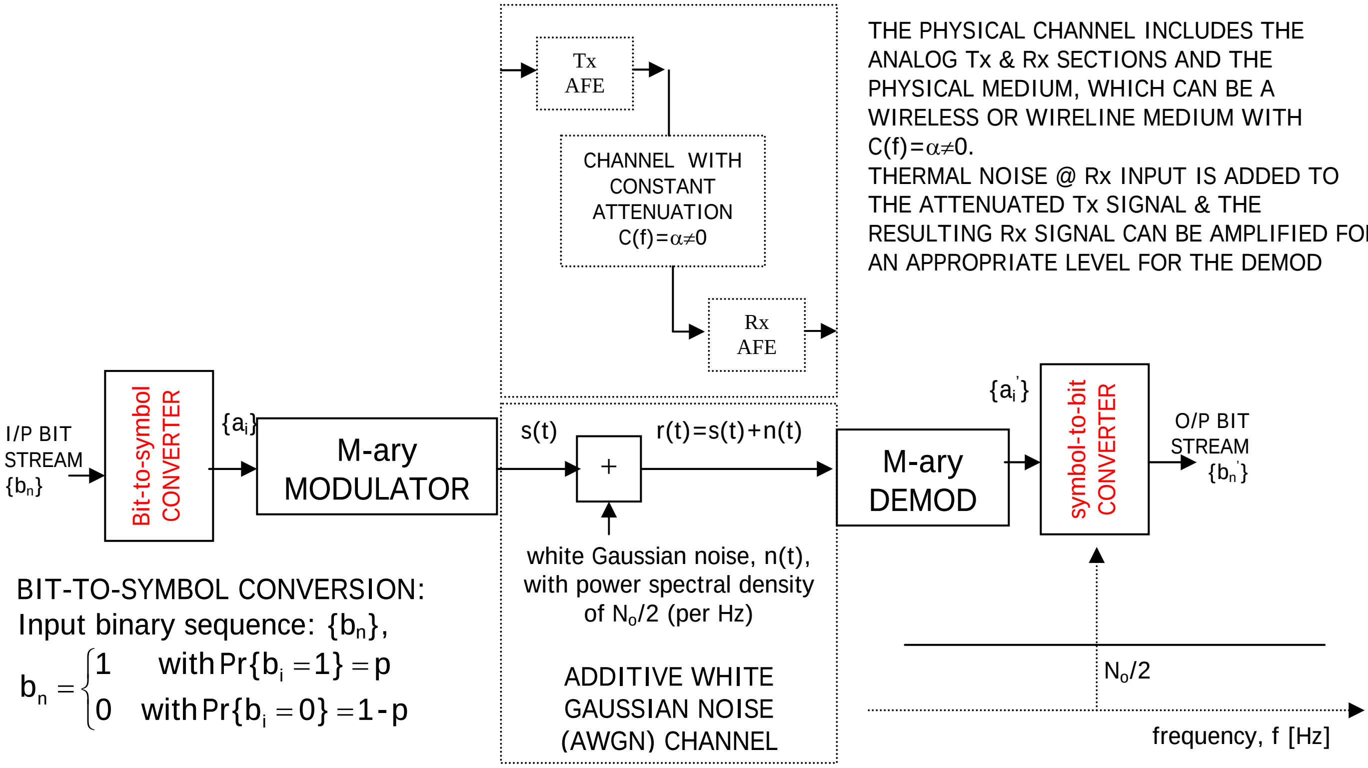
BANDLIMITED SIGNALING OVER AWGN CHANNELS



THE PHYSICAL CHANNEL INCLUDES THE ANALOG Tx & Rx SECTIONS AND THE PHYSICAL MEDIUM, WHICH CAN BE A WIRELESS OR WIRELINE MEDIUM WITH C(f)=α≠0.

THERMAL NOISE @ Rx INPUT IS ADDED TO THE ATTENUATED Tx SIGNAL & THE RESULTING Rx SIGNAL CAN BE AMPLIFIED FOR AN APPROPRIATE LEVEL FOR THE DEMOD

BIT-TO-SYMBOL CONVERSION:
Input binary sequence: $\{b_n\}$,

$$b_n = \begin{cases} 1 & \text{with } \Pr\{b_i = 1\} = p \\ 0 & \text{with } \Pr\{b_i = 0\} = 1-p \end{cases}$$

POWER SPECTRA OF LINEARLY MODULATED SIGNALS

Bandpass signal $s(t) = \mathrm{Re}\left\{v(t)e^{j\omega_c t}\right\}$,

v(t): complex baseband signal, ω_c: center (carrier) frequency

Power spectral density (psd): $S_s(f) = \frac{1}{2}\left[S_v(f - f_c) + S_v(-f - f_c)\right]$

Consider a complex baseband signal $v(t) = \sum_{n=-\infty}^{\infty} I_n g(t - nT)$

g(t): Tx sprectrum-shaping pulse, G(f): Fourier transform of g(t)

I_n: symbol, **real** for M-PAM, **complex** for M-PSK, M-QAM, M-APK

I_n: wide-sense stationary with mean μ_i $\Rightarrow E\{v(t)\} = \mu_i \sum_{n=-\infty}^{\infty} g(t - nT)$

and, autocorrelation function: $\Phi_{II}(m) = \frac{1}{2} E\left\{I_n^* I_{n+m}\right\}$

PAM: $\Phi_{II}(m) = \begin{cases} \mu_i^2 & \text{for } m \neq 0 \\ \mu_i^2 + \sigma_i^2 & \text{for } m = 0 \end{cases}$ for $\{I_n\}$: real, mutually uncorrelated

autocorrelation function of v(t):

$$\Phi_{vv}(t+\tau,t)=\frac{1}{2}E\{v^*(t)v(t+\tau)\}=\frac{1}{2}\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\underbrace{E\{I_n^* I_{m+n}\}}_{\Phi_{II}(m)}g^*(t-nT)g(t+\tau-mT-nT)$$

$$=\sum_{m=-\infty}^{\infty}\Phi_{II}(m).\underbrace{\left[\sum_{n=-\infty}^{\infty}g^*(t-nT)g(t-nT-mT+\tau)\right]}_{\gamma(t+\tau-mT):\ \text{Periodic in t with period T}}$$

$\Phi_{vv}(t+\tau;t)$: **periodic in t** with period T, E{v(t)}: **periodic** with period T

⇒ *v(t)*: **cyclostationary** (periodically stationary in wide sense)

averaging $\Phi_{vv}(t+\tau;t)$ over a single period to remove t

$$\overline{\Phi}_{vv}(\tau)=\frac{1}{T}\int_{-T/2}^{T/2}\Phi_{vv}(t+\tau,t)dt=\sum_{m=-\infty}^{\infty}\Phi_{\text{II}}(m).\frac{1}{T}\sum_{n=-\infty}^{\infty}\int_{-nT-T/2}^{-nT+T/2}g^*(u)g(u+\tau-mT)du$$

$$=\frac{1}{T}\sum_{m=-\infty}^{\infty}\Phi_{\text{II}}(m)\underbrace{\int_{-\infty}^{+\infty}g^*(u)g^*(u+\tau-mT)du}$$

(time-autocorrelation function of g(t):

$\Phi_{gg}(\tau-mT)\xrightarrow{\Im}|G(f)|^2e^{-jm\varpi T}$)

$$\xrightarrow{\Im(\overline{\Phi}_{vv}(\tau))}S_v(f)=\frac{1}{T}|G(f)|^2\sum_{m=-\infty}^{+\infty}\Phi_{II}(m)e^{-j\omega mT}$$

PAM: $\Phi_{II}(m)=\begin{cases}\mu_i^2 & \text{for m}\neq 0\\ \mu_i^2+\sigma_i^2 & \text{for m}=0\end{cases}$ ⇒ $S_v(f)=\frac{1}{T}|G(f)|^2\left\{\sigma_I^2+\mu_I^2\underbrace{\sum_{m=-\infty}^{+\infty}e^{-j\omega mT}}_{\text{periodic in f with period of 1/T}}\right\}$

$$\sum_{m=-\infty}^{+\infty}e^{-j\omega mT}=\frac{1}{T}\sum_{m=-\infty}^{+\infty}\delta(f-\frac{m}{T})$$

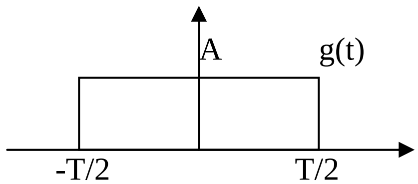
$$S_v(f)=\underbrace{\frac{\sigma_I^2}{T}|G(f)|^2}_{\text{Continuous spectrum}}+\underbrace{\frac{\mu_I^2}{T^2}\sum_{m=-\infty}^{+\infty}\left|G(\frac{m}{T})\right|^2\delta\left(f-\frac{m}{T}\right)}_{\text{Discrete spectral lines}}$$

To remove the discrete spectral lines, we need

- μ_I=0 zero mean sequence or:
- $\left|G\left(\frac{m}{T}\right)\right|^2=0$ for all m

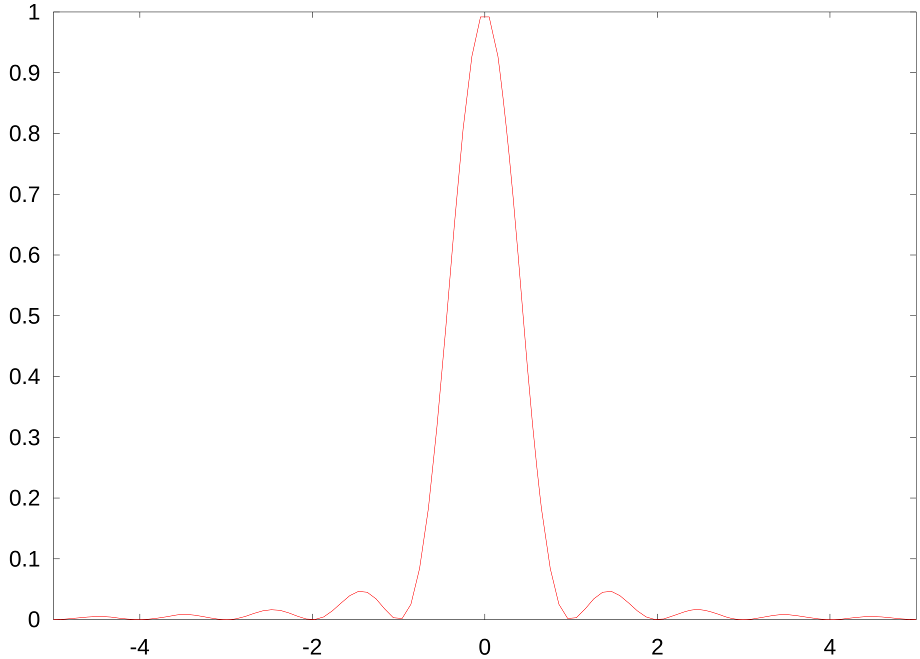
TIME-LIMITED g(t) ⇒ G(f) with INFINITE BW

EXAMPLE: PAM signal using rectangular pulse:

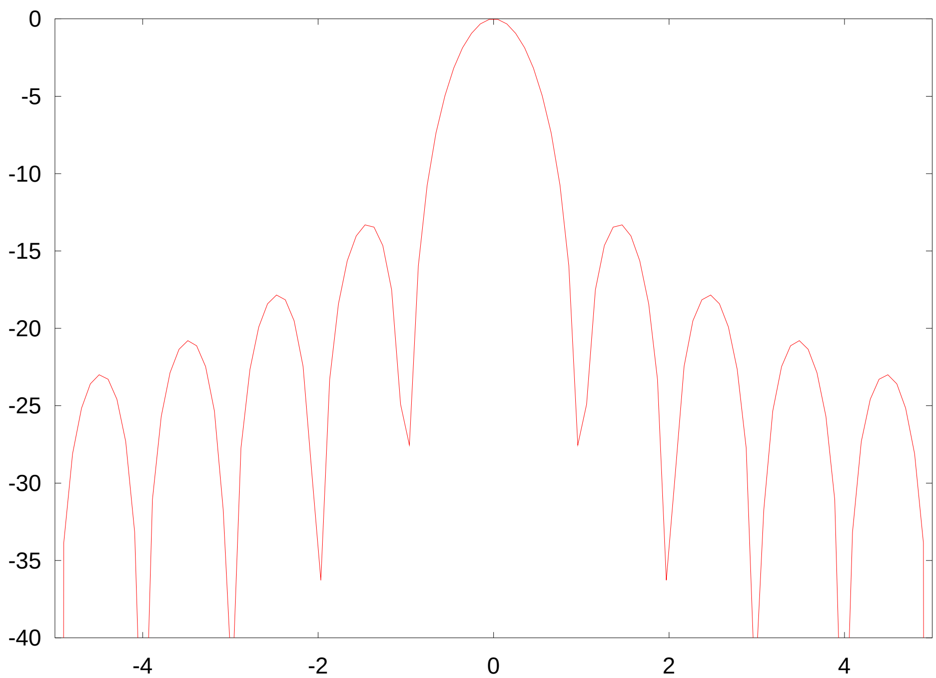


$$G(f) = \int_{-T/2}^{+T/2} A e^{-j2\pi ft} dt = \left. \frac{A}{-j2\pi f} \right|_{t=-T/2}^{t=+T/2}$$

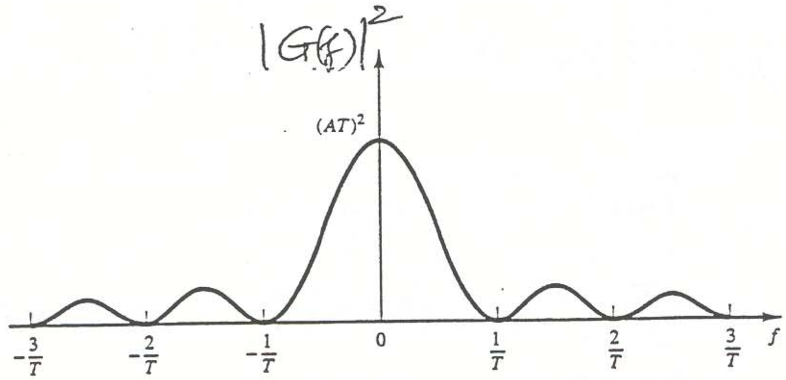
$$= \frac{AT \sin \pi fT}{\pi fT} = AT\text{sinc}(fT)$$

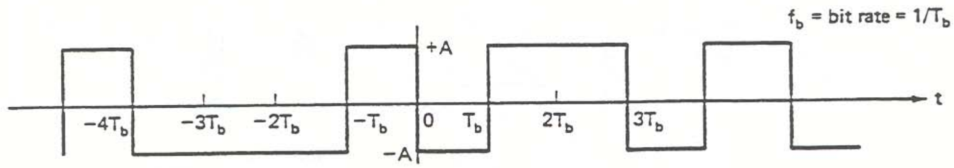


$$\left|\text{sinc}(fT)\right|^2$$

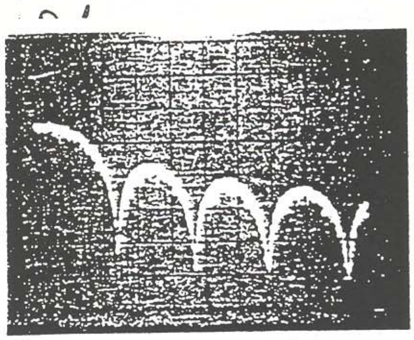


$$20\log_{10}\left|\text{sinc}(fT)\right|$$





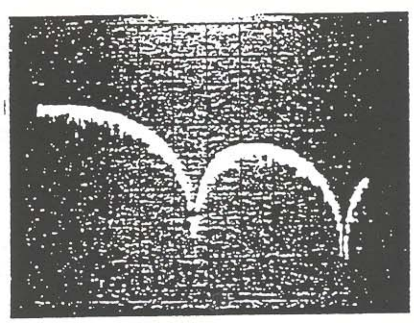
Balanced non-return to zero (NRZ) random-data pattern



(a) H: 20 MHz/div., V: 10 dB/div.

Infinite bandwidth.

45Mb/s

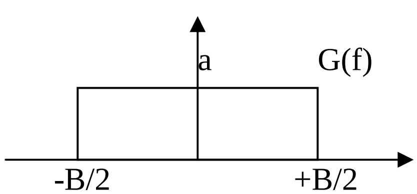


(b) H: 10 MHz/div., V: 10 dB/div.

Infinite bandwidth same as photograph (a but expanded to 10 MHz/div.

⇒ FOR BANDLIMITED TRANSMISSION, WE WANT G(f) with LIMITED BANDWIDTH

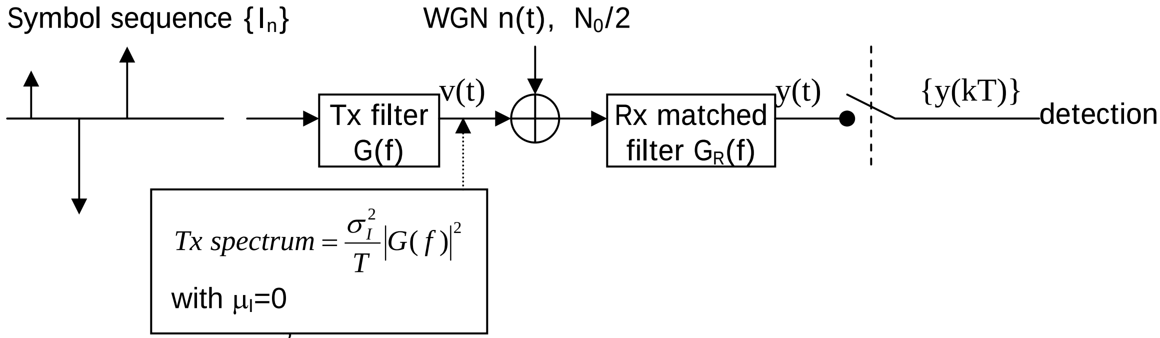
⇒ g(t) is **no longer** time-limited



$$g(t) = \int_{-B/2}^{+B/2} a e^{+j2\pi f t}\, df = \left. \frac{a}{j2\pi t} \right|_{t=-B/2}^{t=+B/2}$$

$$= \frac{aB \sin \pi B t}{\pi B t} = aB \operatorname{sinc}(Bt)$$

Tx AND Rx PARTITIONING:



Tx $P_{avg}=E\{|v(t)|^2\}=\frac{\sigma_I^2}{T}\int_{-\infty}^{+\infty}|G(f)|^2 df \Rightarrow$ Tx $E_s=T\,P_{avg}=\sigma_I^2\int_{-\infty}^{+\infty}|G(f)|^2 df$

Define $h(t)=g(t)*g_R(t)$, at the Rx filter output:

$$y(t)=[v(t)+n(t)]\otimes g_R(t)=\sum_{n=-\infty}^{\infty} I_n h(t-nT)+n(t)\otimes g_R(t)$$

$$y(kT)=\sum_{n=-\infty}^{\infty} I_n h[(k-n)T]+n_k=\underbrace{I_k h(0)}_{\text{Main sample}}+\underbrace{\sum_{\substack{m=-\infty \\ m\neq 0}}^{+\infty} I_{k-m}h(mT)}_{\text{Intersymbol inteference}}+\underbrace{n_k}_{\text{G. noise}}$$

n_k: Gaussian noise with zero mean and variance: $\sigma_n^2=\frac{N_o}{2}\int_{-\infty}^{+\infty}|g_R(t)|^2 dt$

RECALL: for **time-limited** g(t), i.e., g(t)=0 for $t\notin[0,T]$, the Rx uses a matched filter: $g_R(t)=g(T-t)$, $\Rightarrow g_R(t)\xrightarrow{\Im} G_R(f)=G^*(f)e^{-j\omega T}$
(THE MATCHED FILTER PROVIDES MAXIMUM output SNR)
$g_R(t)=0$ for $t\notin[0,T] \Rightarrow h(t)=g(t)*g_R(t)=0$ for $t\notin[-T,T]$ (or $[0,2T]$)
$\Rightarrow$ the **intersymbol interference (ISI)** term is 0, i.e., **no ISI**

For **bandlimited** signaling, $g(t)\neq 0$ for $t\notin[0,T]$. However, the **ISI** term can be eliminated when $h(mT)=0$ for **all** $m\neq 0$
$\Rightarrow$ WE WANT BANDLIMITED G(f), $G_R(f)=G^*(f)e^{-j\omega T}$ AND $H(f)=G(f)G_R(f)$ WITH $h(mT)=0$ for **all** $m\neq 0$

DERIVATION: MATCHED FILTER & MAXIMUM output SNR:

$y(t)= v(t)*g_R(t)+n(t)*g_R(t)$

power of noise part $n(t)*g_R(t)$:

$$N=\int_{-\infty}^{+\infty}\frac{N_o}{2}|G_R(f)|^2 df = \frac{N_o}{2}\int_{-\infty}^{+\infty}G_R(f)G_R^*(f)df = \frac{N_o}{2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}G_R^*(f)g_R(t)e^{-j2\pi ft}dtdf$$

$$N=\frac{N_o}{2}\int_{-\infty}^{+\infty}g_R(t)g_R^*(t)dt = \frac{N_o}{2}\int_{-\infty}^{+\infty}|g_R(t)|^2 dt$$

signal part: $a(t)=v(t)*g_R(t)=\sum_{n=-\infty}^{\infty}I_n h(t-nT)$ where $h(t)= g(t)*g_R(t)$

$$a(kT)=\sum_{n=-\infty}^{+\infty}I_n\int_{-\infty}^{+\infty}g(\tau)g_R([k-n]T-\tau)d\tau = I_k\int_{-\infty}^{+\infty}g(\tau)g_R(-\tau)d\tau$$

for time-limited g(t), i.e., g(t)=0 for $t\notin[0,T]$

Signal power: $E\{a^2(kT)\}= \sigma_I^2\left|\int_{-\infty}^{+\infty}g(t)g_R(-t)dt\right|^2, \quad \sigma_I^2 = E\{|I_k|^2\}$

Output signal-to-noise power ratio: $SNR=\frac{\sigma_I^2}{N_o/2}\frac{\left|\int_{-\infty}^{+\infty}g(t)g_R(-t)dt\right|^2}{\int_{-\infty}^{+\infty}|g_R(t)|^2 dt}$

is maximum when $g_R(t)=g(T-t)$ or $G_R(f)=G^*(f)e^{-j\omega T}$

note: The Cauchy-Schwarz inequality: $\left|\int_{-\infty}^{+\infty}x(t)y^*(t)dt\right|^2 \le \int_{-\infty}^{+\infty}|x(t)|^2 dt\int_{-\infty}^{+\infty}|y(t)|^2 dt$

and the equality holds when y(t)=Cx(t), C: any constant

BANDLIMITED SIGNALING IN AWGN WITH ZERO ISI

$\Rightarrow$ zero ISI when: $h(mT)=\begin{cases}h_0 & m=0\\ 0 & m\neq 0\end{cases}$ in time domain

$\Rightarrow$ Nyquist theorem: H(f) satisfies $\sum_{m=-\infty}^{\infty}H\left(f+\frac{m}{T}\right)=h_0\mathrm{T}:\text{constant}$

in frequency domain

DERIVATION:

$$h(mT) = \int_{-\infty}^{+\infty} H(f) e^{j2\pi mTf} df = \sum_{k=-\infty}^{+\infty} \int_{(2k-1)/2T}^{(2k+1)/2T} H(f) e^{j2\pi mTf} df = \sum_{k=-\infty}^{+\infty} \int_{-1/2T}^{+1/2T} H(\lambda + \frac{k}{T}) e^{j2\pi mT\lambda} d\lambda$$

$$= \int_{-1/2T}^{+1/2T} Z(\lambda) e^{j2\pi mT\lambda} d\lambda, \quad with \quad Z(\lambda) = \sum_{k=-\infty}^{+\infty} H(\lambda + \frac{k}{T})$$

$Z(\lambda)$ is periodic function of λ with period 1/T, hence can be represented in a Fourier series, i.e., $Z(\lambda) = \sum_{n=-\infty}^{+\infty} z_n e^{j2\pi nT\lambda}$ with $z_n = T\int_{-1/2T}^{1/2T} Z(\lambda) e^{-j2\pi nT\lambda} d\lambda$

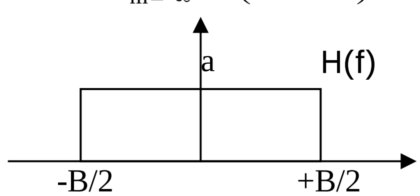
$\Rightarrow$ $h(mT)=z_{-m}/T$, i.e.,

for $h(mT) = \begin{cases} h_0 & m=0 \\ 0 & m \neq 0 \end{cases}$ we need $Z(\lambda) = \sum_{k=-\infty}^{\infty} H\left(\lambda + \frac{k}{T}\right) = Th_0 : \text{constant}$

Consider simple, ideal, strictly bandlimited $H(f) = \begin{cases} H_0 & \text{for} \left| f \right| \leq \frac{B}{2} \\ 0 & \text{for} \left| f \right| > \frac{B}{2} \end{cases}$

what is the narrowest bandwidth (B) such that $h(mT)=0$ for $m \neq 0$?

or H(f) satisfies $\sum_{m=-\infty}^{\infty} H\left(f + \frac{m}{T}\right) = T \text{ constant}$

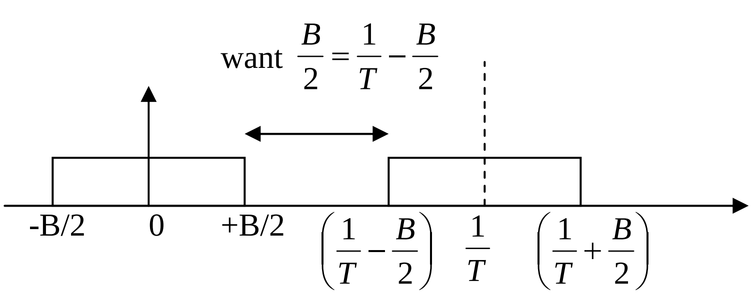


$$h(t) = \int_{-B/2}^{+B/2} a e^{+j2\pi ft} df = \left. \frac{a}{j2\pi t} \right|_{t=-B/2}^{t=+B/2}$$

$$= \frac{aB \sin \pi Bt}{\pi Bt} = aB \text{sinc}(Bt)$$

$\Rightarrow$ time-domain: $h(0)=aB$, $h(m/B)=0$

frequency-domain:



$\Rightarrow$B=1/T: **minimum** BW for **zero ISI**

$\Rightarrow$ spectral efficiency:

1 symbol/s/Hz = m b/s/Hz for linear modulation $M=2^m$

BUT THE IDEAL "BRICK-WALL" FILTER WITH RECTANGULAR FREQUENCY RESPONSE IS NOT PHYSICALLY REALIZABLE!

raised-cosine filter

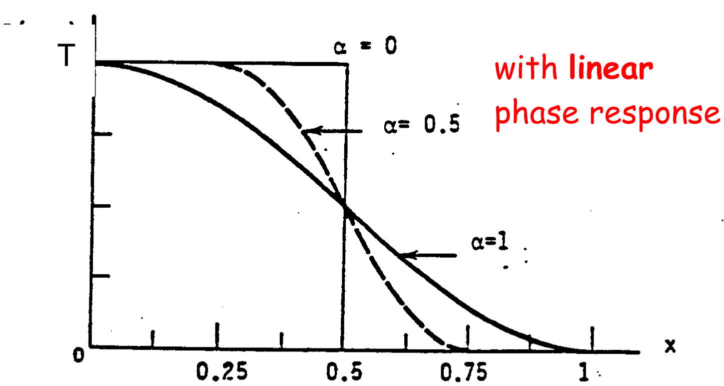
TRANSFER FUNCTION:

$$H(f)=\begin{cases} T & 0\le |f| \le \frac{1}{2T}(1-\alpha) \\ \frac{T}{2}\left\{1+\cos\left[\frac{\pi T}{\alpha}\left(|f|-\frac{1-\alpha}{2T}\right)\right]\right\} & \frac{1-\alpha}{2T}\le |f| \le \frac{1+\alpha}{2T} \\ 0 & |f| > \frac{1+\alpha}{2T} \end{cases}$$

or:

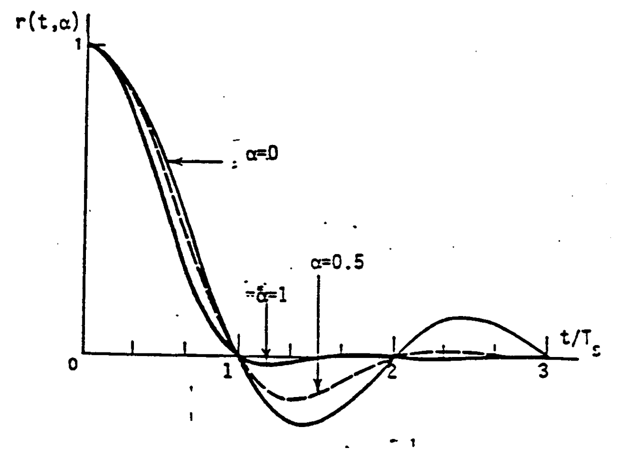
$$R(x,\alpha)=\begin{cases} T & 0\le x\le (1-\alpha)/2 \\ \cos^2\{\pi[x-\{(1-\alpha)/2\}]/2\alpha\} & (1-\alpha)/2\le x\le (1+\alpha)/2 \\ 0 & elsewhere \end{cases}$$

where $x=(f-f_c)T$



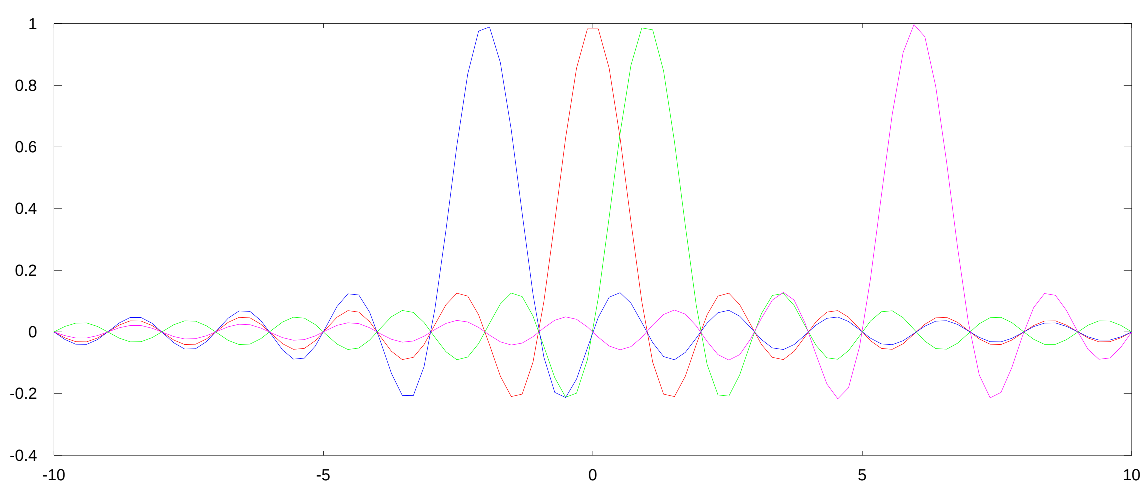
IMPULSE RESPONSE:

$$\Rightarrow r(t,\alpha)=\frac{\cos\left(\frac{\pi\alpha t}{T_S}\right)}{1-\left(\frac{2\alpha t}{T_S}\right)^2}\operatorname{sinc}(\pi t/T_S)$$

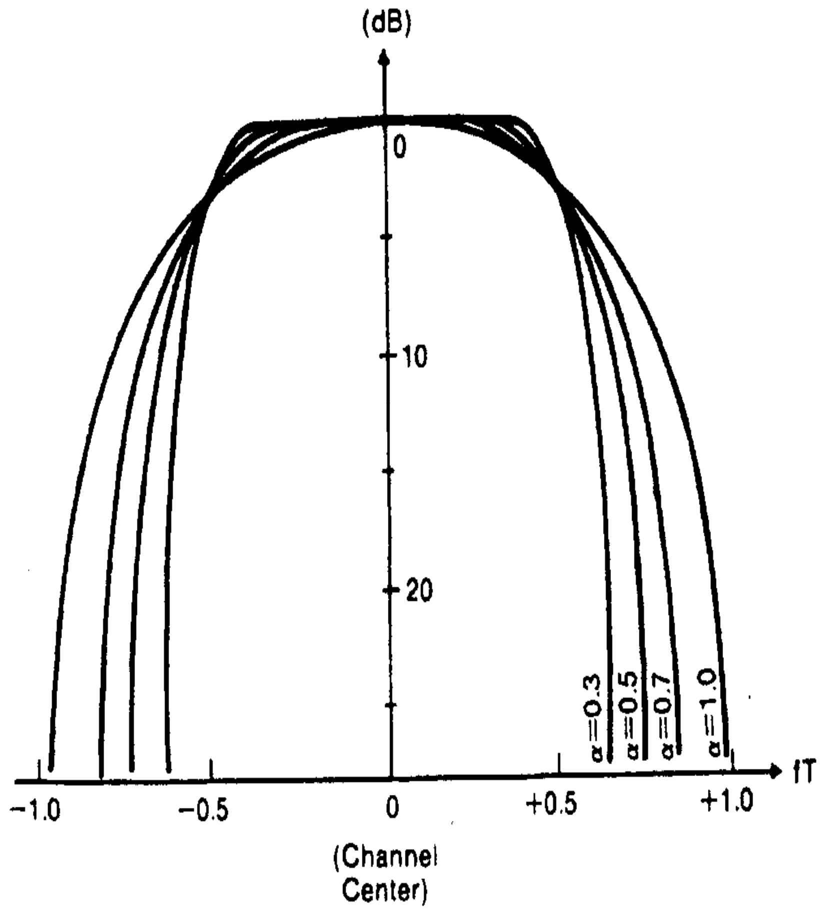


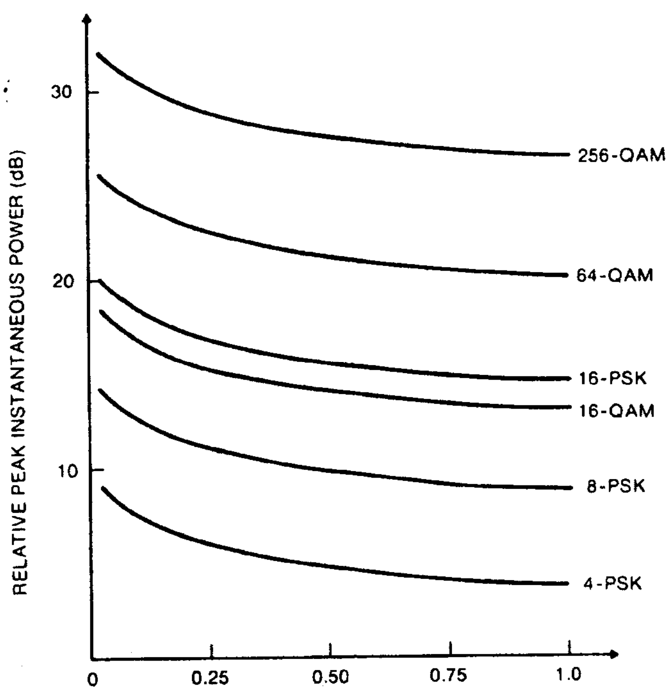
Responses of a raised-cosine filter to an input sequence of:

$$\partial(t), \partial(t-2T), \partial(t-3T), \partial(t-8T)$$



SPECTRA OF Tx SIGNALS USING RRC FILTERS:

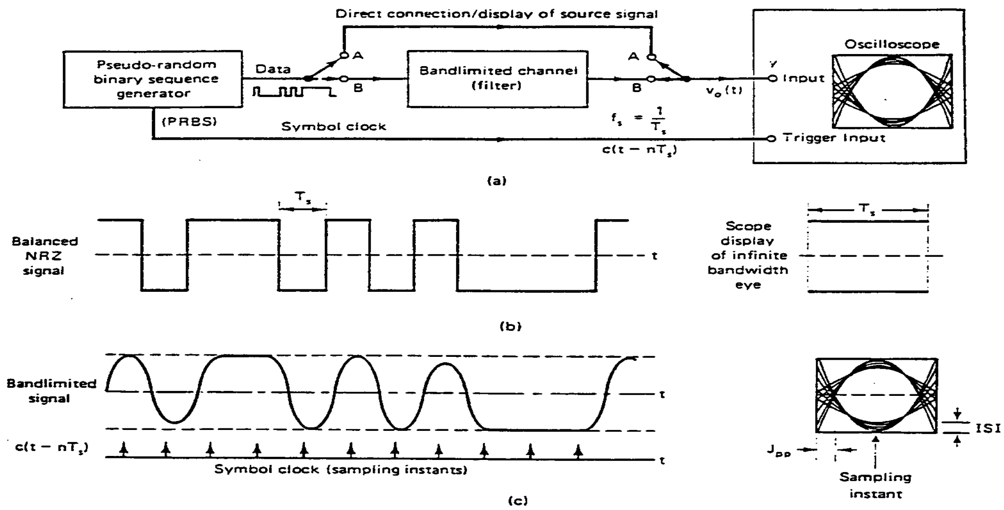




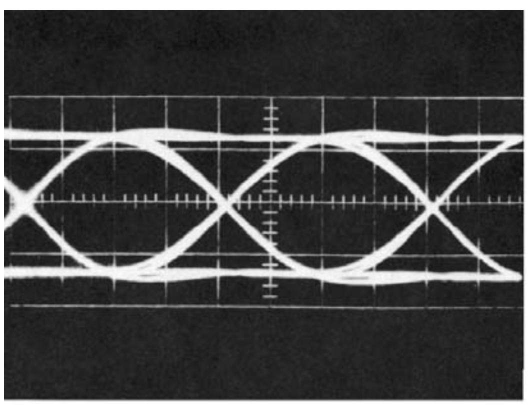
PEAK INSTANTANEOUS POWER RELATIVE TO POWER OF A QPSK SIGNAL USING TIME-LIMITED RECTANGULAR PULSE SHAPE

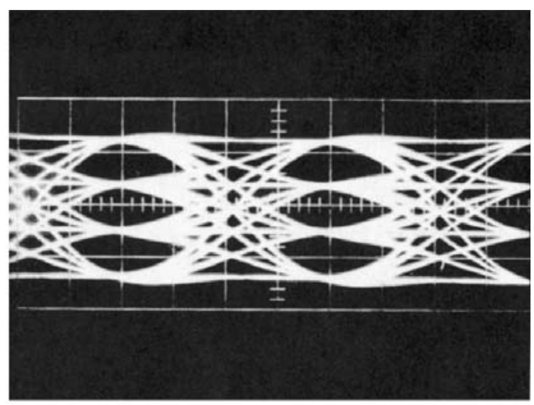
(from T. Noguchi, Y. Daido, J.A. Nossek, "Modulation Techniques for Microwave Digital Radio", *IEEE Communications Magazine*, October 1986, pp. 21-30)

EYE DIAGRAMS:

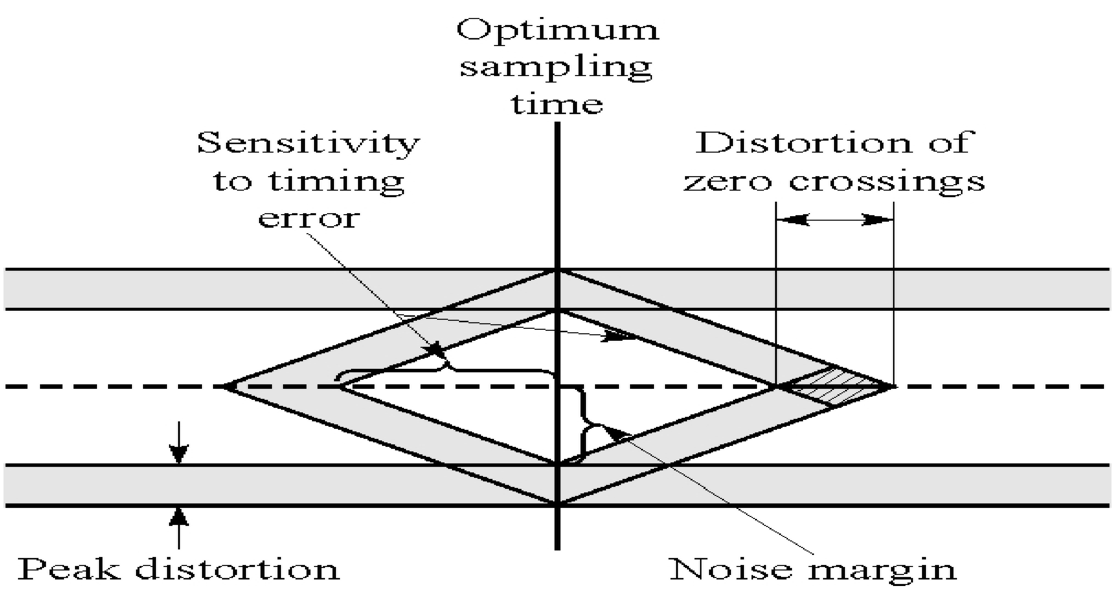


Examples of eye patterns for

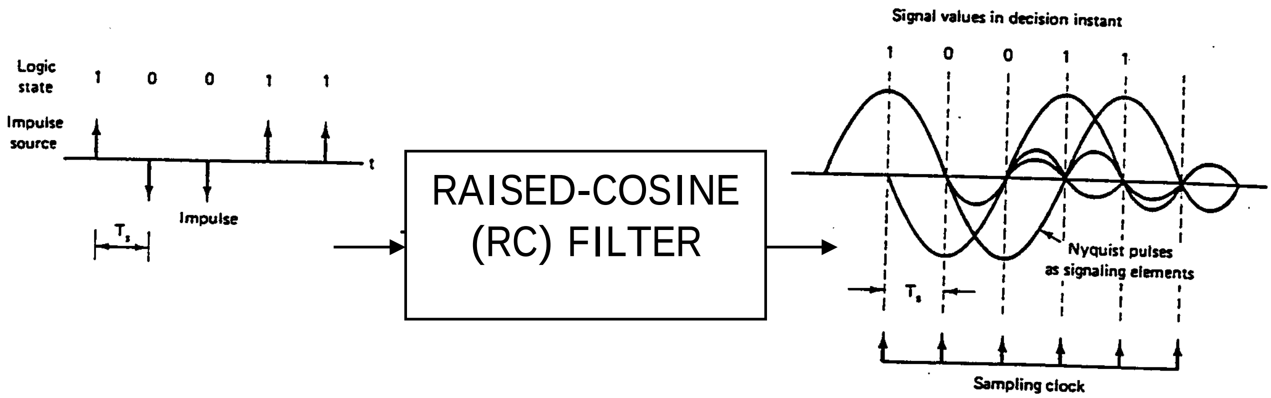


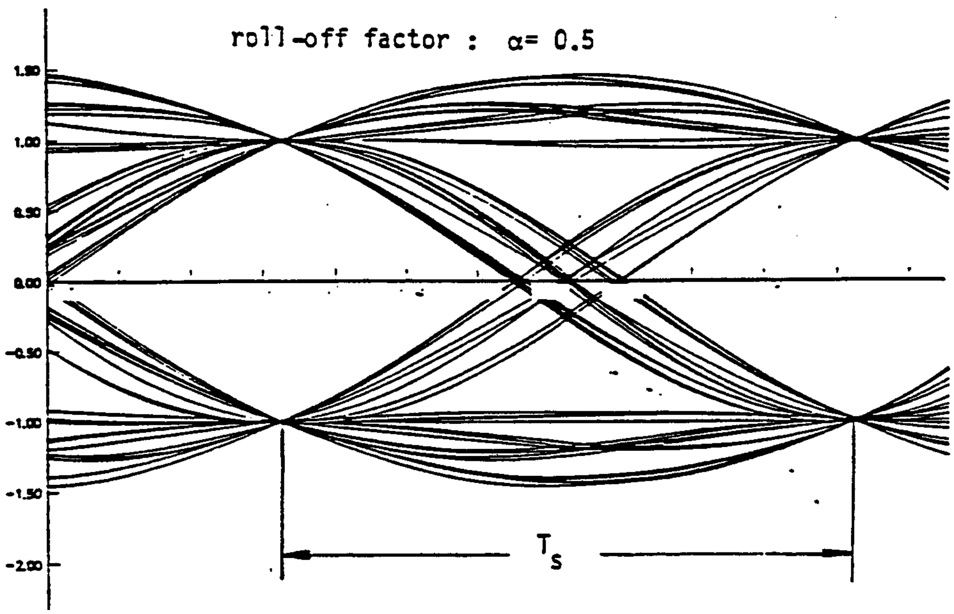


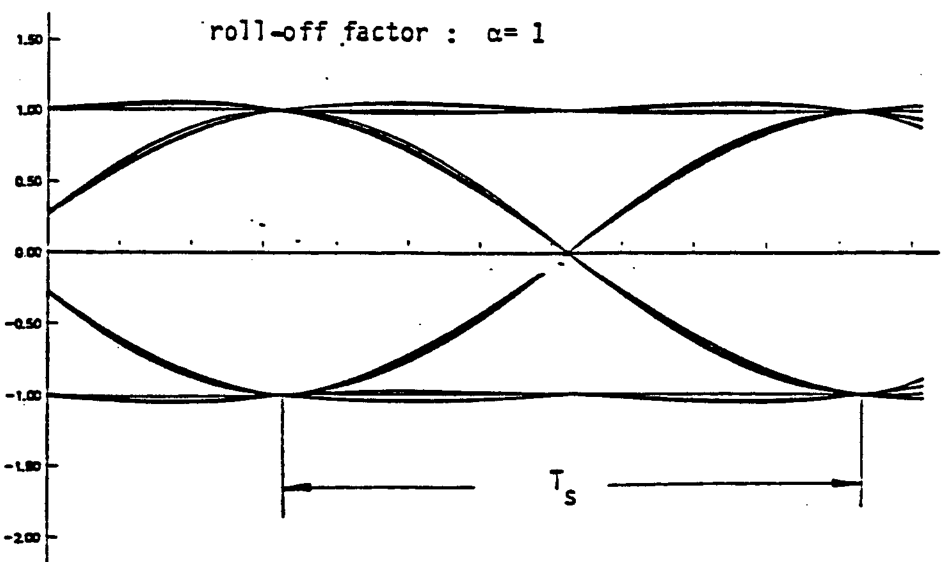
2-PAM 4PAM



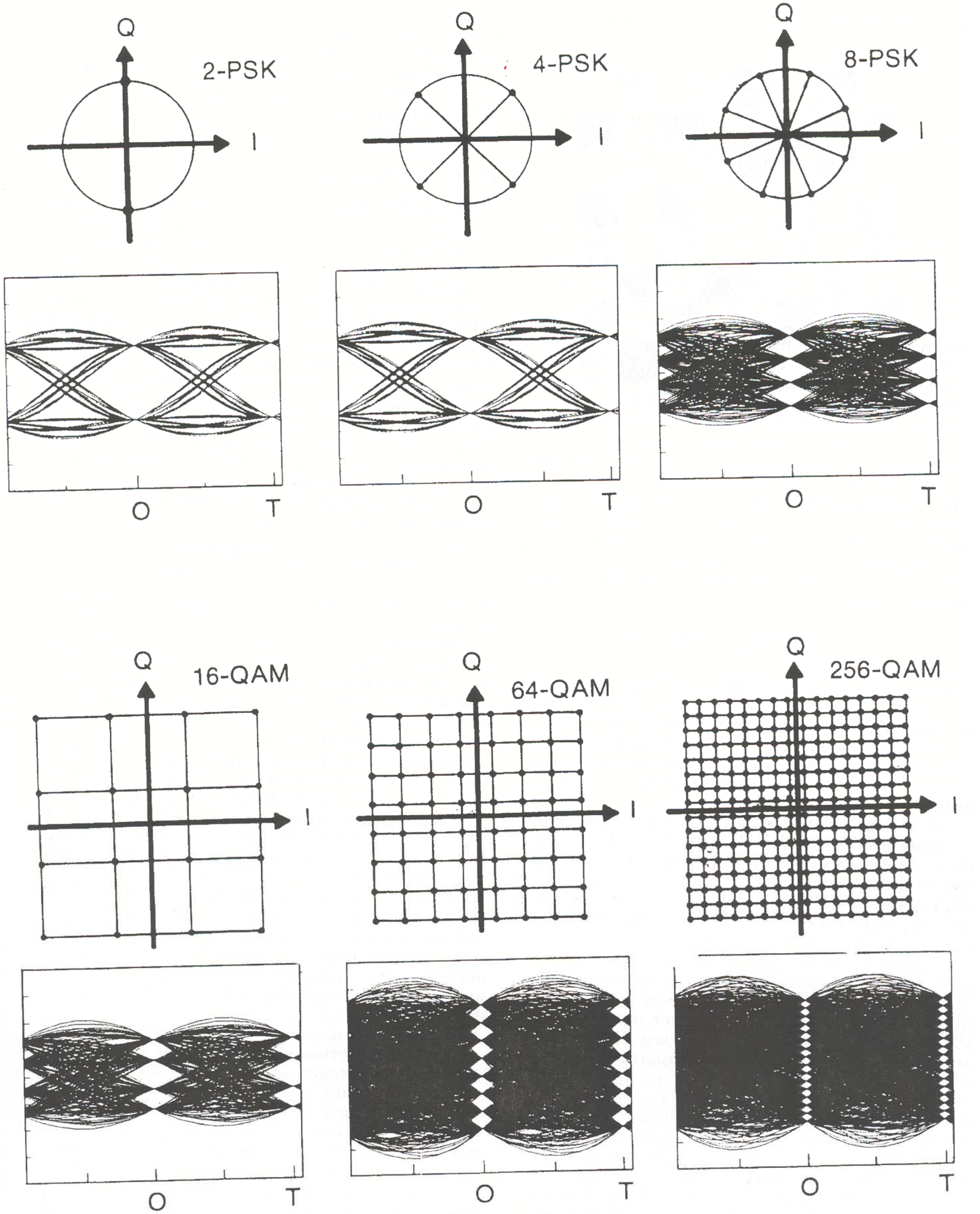
EYE DIAGRAMS FOR RAISED-COSINE FILTER:



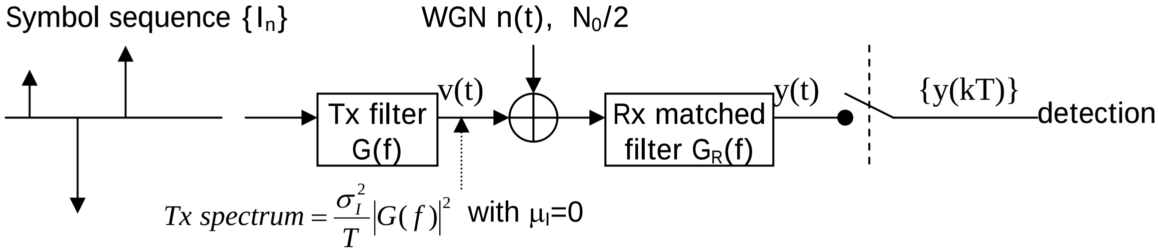




EYE DIAGRAMS FOR RAISED-COSINE FILTER:



DESIGN OF Tx AND Rx FILTERS FOR BANDLIMITED TRANSMISSION IN AWGN CHANNELS WITH ZERO ISI



For maximum output SNR, select: $G_R(f)=G^*(f)e^{-j\omega T}$

For zero ISI in y(kT), select: $H(f)=G_R(f)G^*(f)=R(f,\alpha)$: raised-cosine (RC)

⇒ optimum partition: linear-phase Tx and Rx filters with **root raised-cosine (RRC)** amplitude response, i.e.,
$G(f)=[R(f,\alpha)]^{1/2}e^{j2\pi f\tau}$ and $G_R(f)=[R(f,\alpha)]^{1/2}e^{j2\pi fd}$
where τ and d: constant **group delay**

Tx signal: $v(t)=\sum_{n=-\infty}^{\infty} I_n g(t-nT)$, avg. Tx power: $P_{avg}=\frac{\sigma_I^2}{T}\int_{-\infty}^{+\infty}|G(f)|^2 df=\frac{\sigma_I^2}{T}\int_{-\infty}^{+\infty}|R(f,\alpha)|df=\frac{\sigma_I^2}{T}$

Avg energy per symbol: $E_s=TP_{avg}=\sigma_I^2$.

Rx sample $y(kT)=I_k+n_k$ since $h_0=h(0)=1$, I_k: signal sample

n_k: Gaussian noise sample with zero mean, variance: $\int_{-\infty}^{+\infty}\frac{N_o}{2}|G_R(f)|^2 df=\frac{N_o}{2}$

$$E\{n_k n_l\}=E\left\{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} w(x)w(y)g_R(kT-x)g_R(lT-y)dxdy\right\}=\frac{N_o}{2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\partial(x-y)g_R(kT-x)g_R(lT-y)dxdy$$

$$=\frac{N_o}{2}\int_{-\infty}^{+\infty} g_R(kT-x)g_R(lT-x)dx=\frac{N_o}{2}\int_{-\infty}^{+\infty} g_R(t)g_R([l-k]T-t)dt=\frac{N_o}{2}h([l-k]T)=\frac{N_o}{2}\partial([l-k]T)$$

Example: bandlimited M-ASK

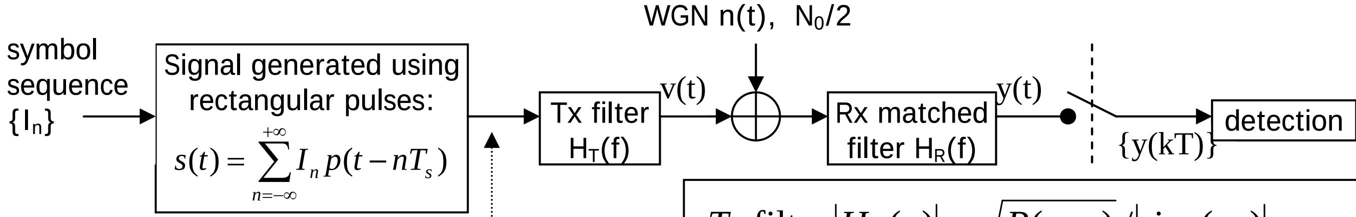
$I_k \in \{A(2n-1-M), n=1,2,...,M\}$ $E_s=TP_{avg}=\sigma_I^2=A^2(M^2-1)/3$

Rx sample $y(kT)=I_k+n_k$

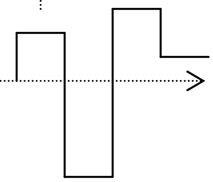
I_k: signal sample with $d_{min}=2A$ or $[d_{min}]^2=12E_s/(M^2-1)$

Performance: same as the time-limited case: $P_e=\left[1-\frac{1}{M}\right]erfc\left[\sqrt{\frac{3}{M^2-1}\frac{E_s}{N_0}}\right]$

OPTIMUM PARTITION OF Tx/Rx FILTERS FOR BANDLIMITED TRANSMISSION IN AWGN CHANNELS WITH ZERO ISI: CASE OF INPUT RECTANGULAR PULSES



$$p(t)=\begin{cases}1, & 0\le t<T_s\\ 0, & elsewhere\end{cases}$$



$$|W_S(f)|=C\mathrm{sinc}^2(\pi f T_S)$$

Tx filter $|H_T(x)|=\sqrt{R(x,\alpha)}\,/|\mathrm{sinc}(\pi x)|$

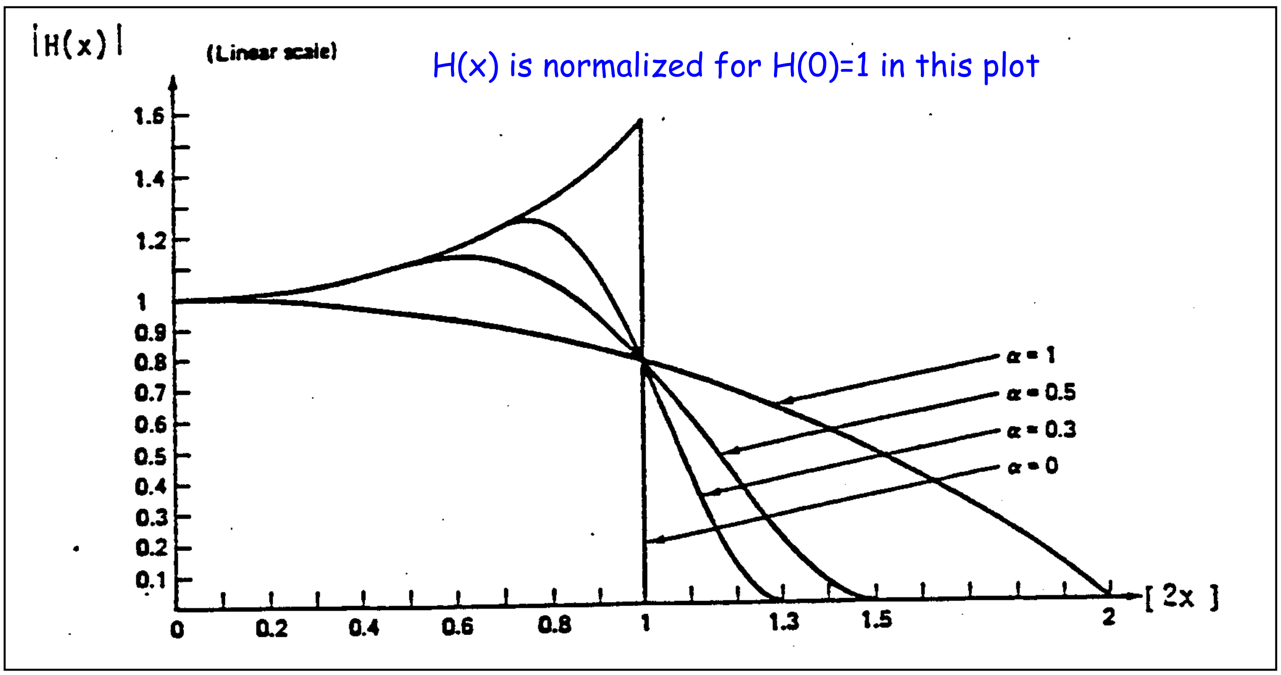
Rcv filter $|H_R(x)|=\sqrt{R(x,\alpha)}$

overall channel transfer function

$|H(x)|=|H_T(x)||H_R(x)|=|R(x,\alpha)/\mathrm{sinc}(\pi x)|$

$x=(f-f_c)T_S$, f_c: carrier frequency

T_S: symbol interval



GENERAL CASE: p(t)↔P(f): Fourier transform of p(t)

Tx filter $|H_T(x)|=\sqrt{R(x,\alpha)}\,/|P(x)|$

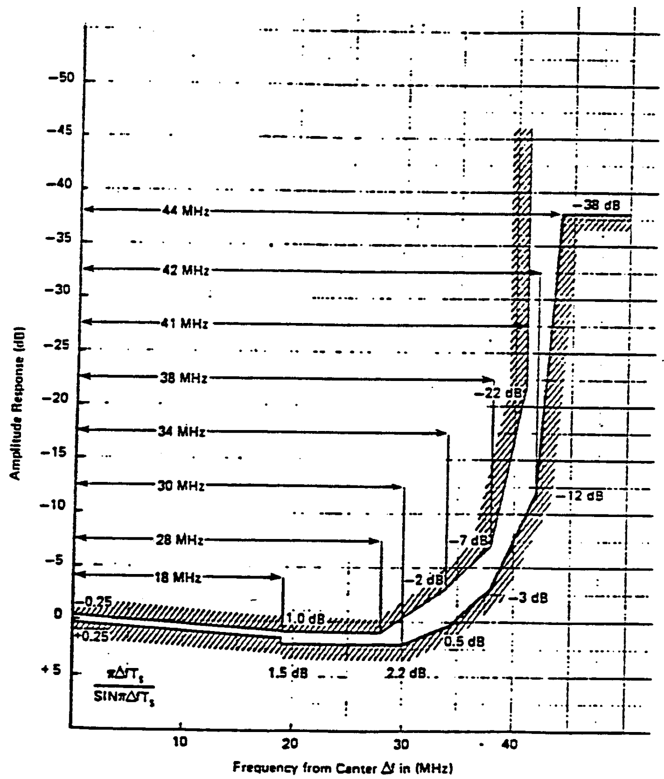
Rcv filter $|H_R(x)|=\sqrt{R(x,\alpha)}$

overall channel transfer function

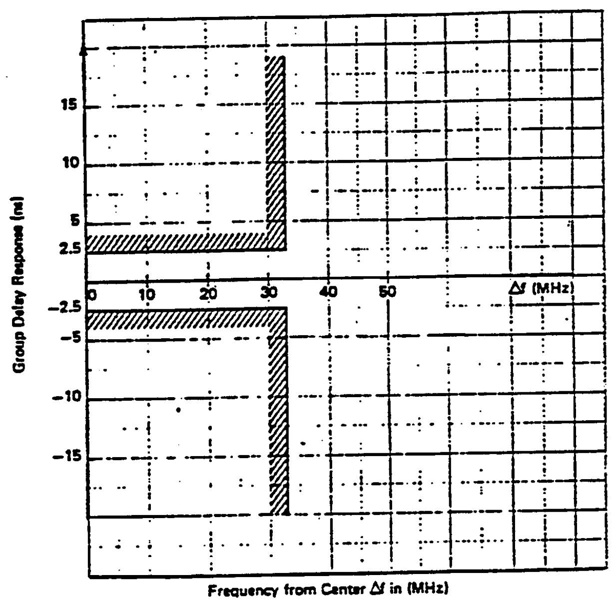
$|H(x)|=|H_T(x)||H_R(x)|=|R(x,\alpha)/P(x)|$

EXAMPLE OF FILTERING REQUIREMENTS: INTELSAT V TDMA/QPSK

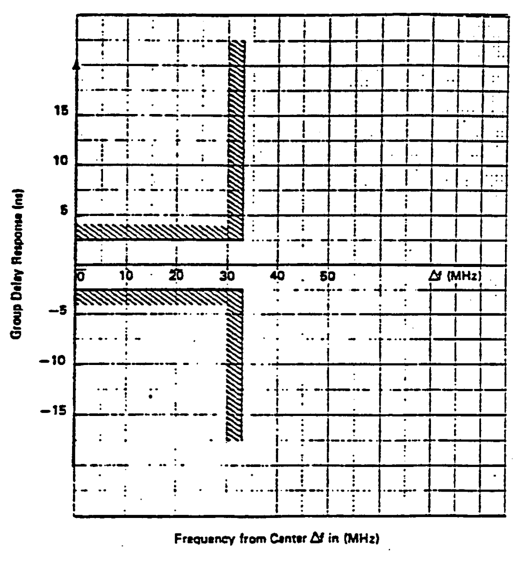
Carrier frequency: f_C=140 MHz Tx bit rate: f_b=120.832 MHz
Nyquist frequency: f_N = f_S/2=30.208 MHz Raised cosine filter with α=0.4



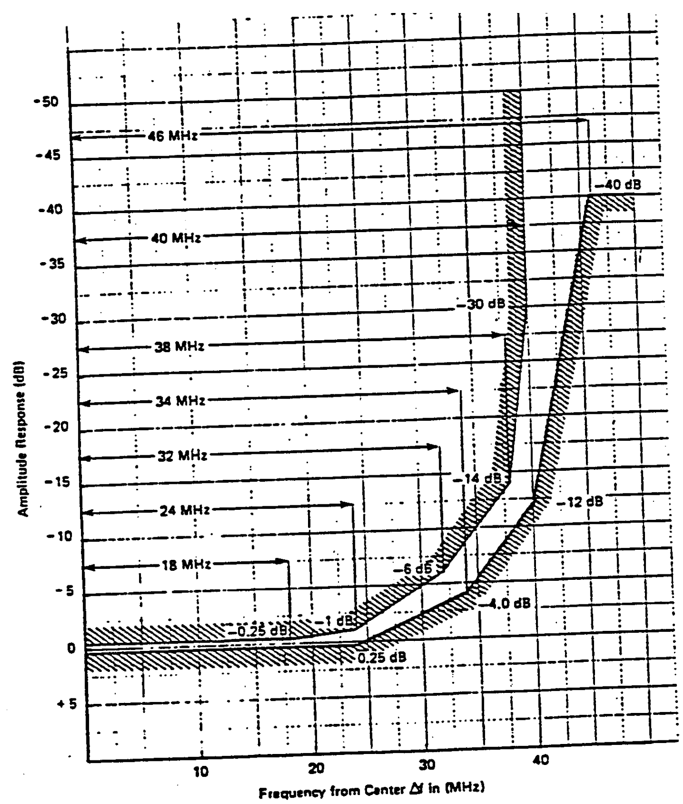
MODULATOR FILTER AMPLITUDE MASK



MODULATOR GROUP DELAY MASK



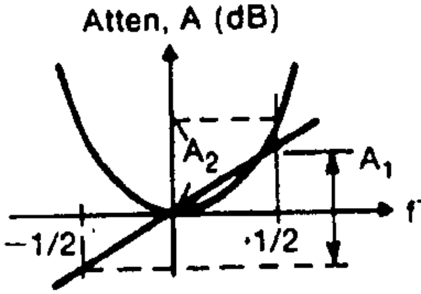
DEMODULATOR FILTER GROUP DELAY MASK



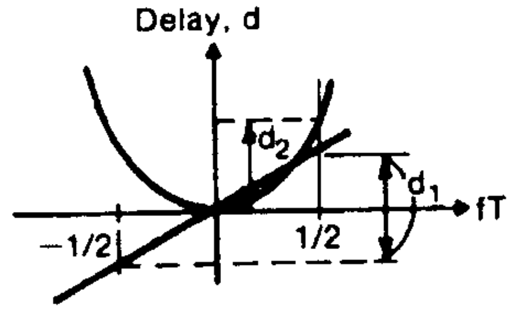
DEMODULATOR FILTER AMPLITUDE MASK

EFFECTS OF AMPLITUDE AND PHASE DISTORTIONS ON RECEIVER PERFORMANCE

(from T. Noguchi, Y. Daido, J.A. Nossek, “Modulation Techniques for Microwave Digital Radio”, *IEEE Communications Magazine*, October 1986, pp. 21-30)



Amplitude Distortion



Delay Distortion