Frequency-Flat Fading Channels & Diversity Techniques

REFERENCES:

A. Goldsmith, *Wireless Communications,* Cambridge University Press, 2005

H. R. Anderson, *Fixed Broadband Wireless System Design*, John Wiley & Sons, 2003

D. Tse, P. Viswanath, *Fundamentals of Wireless Communication,* Cambridge University Press, 2005

Marvin K. Simon and Mohamed-Slim Alouini, *Digital Communication over Fading Channels, 2nd Edition,* John Wiley & Sons, 2005

J. Mark, W. Zhuang, *Wireless Communications and Networking*, Prentice-Hall, 2003

P.M. Shankar, *Introduction to Wireless Systems*, John Wiley & Sons, 2002

T.S. Rappaport, *Wireless Communications: Principles and Practice*, 2nd Ed, Prentice-Hall, 2002

- and materials from various sources

Performance of M-ary Digital Modulation in an AWGN Channel

binary: M = 2, $P_b = P_e = \frac{1}{2} erfc\left[\frac{d_{12}}{2\sqrt{N_0}}\right] = \frac{1}{2} erfc\left[\sqrt{\frac{E_b}{N_0}(1-\gamma_{12})}\right]$

$M-ary: E_S = \left(\log_2 M\right) E_b$

Union bound: $P_e \le \frac{1}{2}(M-1) erfc\left[\frac{d_{min}}{2\sqrt{N_0}}\right]$

M-ary ASK: $P_e \approx erfc\sqrt{\frac{3}{M^2-1}\frac{E_S}{N_0}}, d_{min} = \sqrt{\frac{12}{(M^2-1)}E_S}$

M-ary PSK: $P_e \approx erfc\left[\sin\frac{\pi}{M}\sqrt{\frac{E_S}{N_0}}\right], d_{min} = \sqrt{E_S}.\sin \pi/M$

squared M-ary QAM: $P_{e,M-aryQAM} \approx 2P_{eASK} \approx 2\left(1-\frac{1}{\sqrt{M}}\right) erfc\left(\sqrt{\frac{3}{2(M-1)}\frac{E_S}{N_0}}\right)$

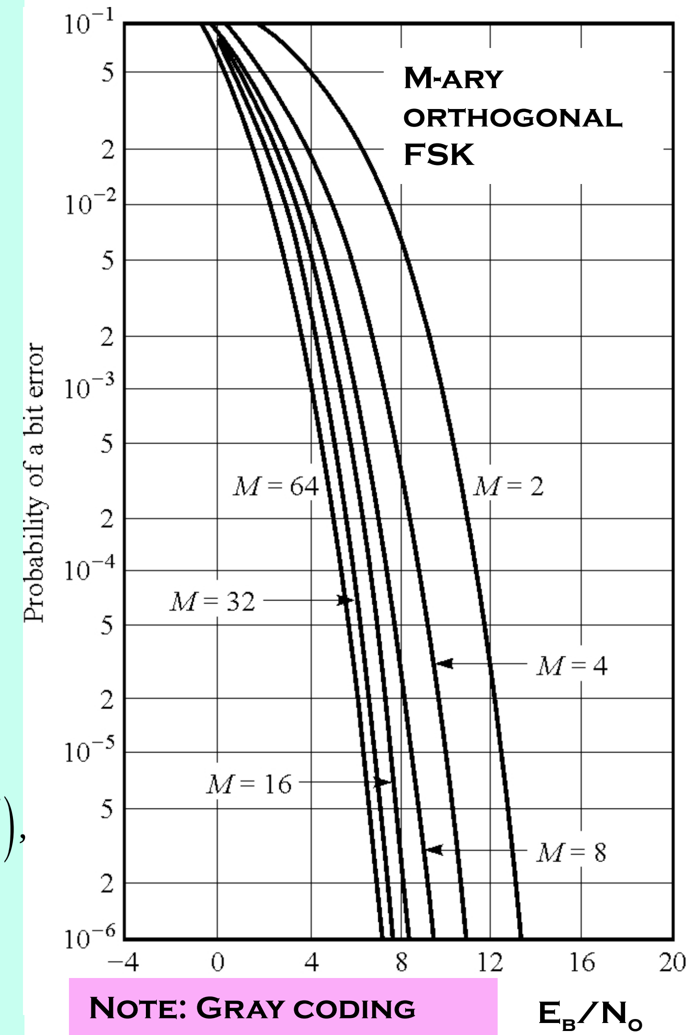
Orthogonal FSK: $P_e \le \frac{1}{2}(M-1) erfc\sqrt{\frac{E_S}{2N_0}}$

$\rightarrow$ General Performance form: $P_e \approx A_M Q\left(\sqrt{B_M^2 (E_s/N_0)}\right) = a_M erfc\left(\sqrt{b_M^2 (E_s/N_0)}\right)$,

$E_s/N_0 = P/(f_s N_0) = SNR$ (f_s: Nyquist BW)

$erfc(x) = 2Q\left(x\sqrt{2}\right)$, where: $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du = \frac{1}{\pi}\int_0^{\pi/2} e^{-x^2/(2\sin^2\theta)} d\theta$

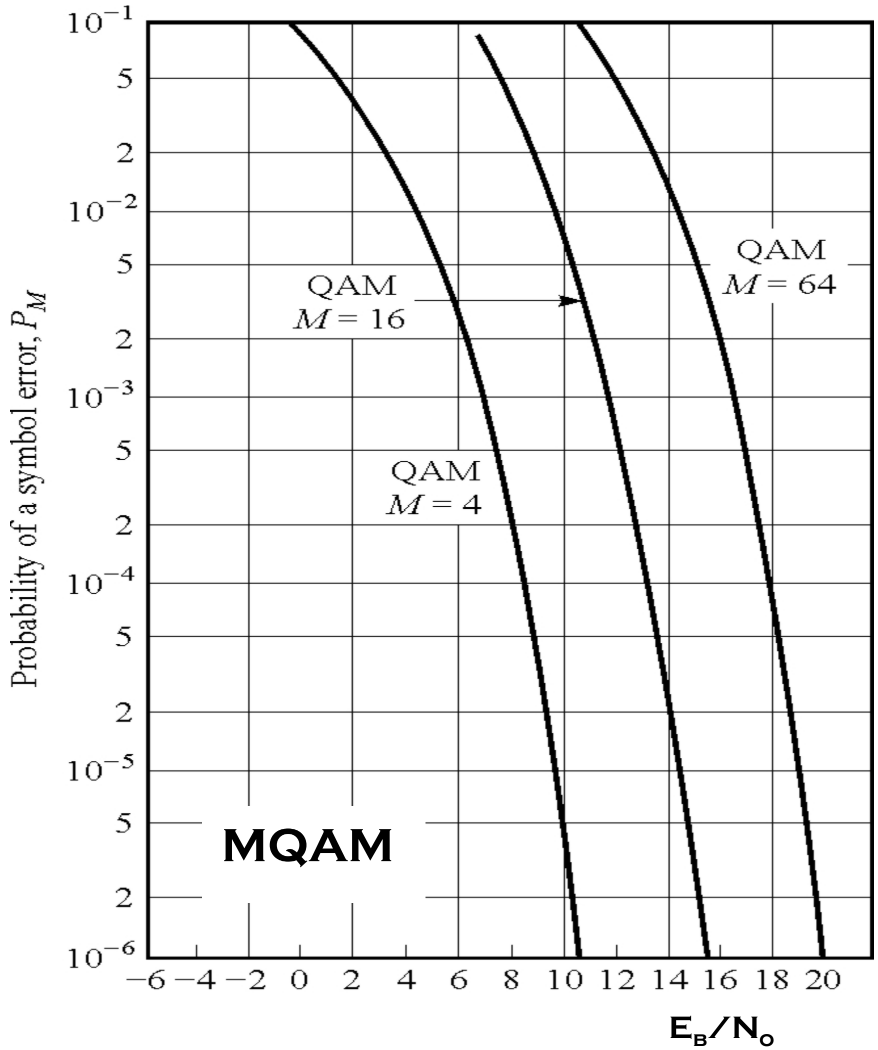
M-ary orthogonal FSK signaling schemes are power-efficient but not bandwidth-efficient.

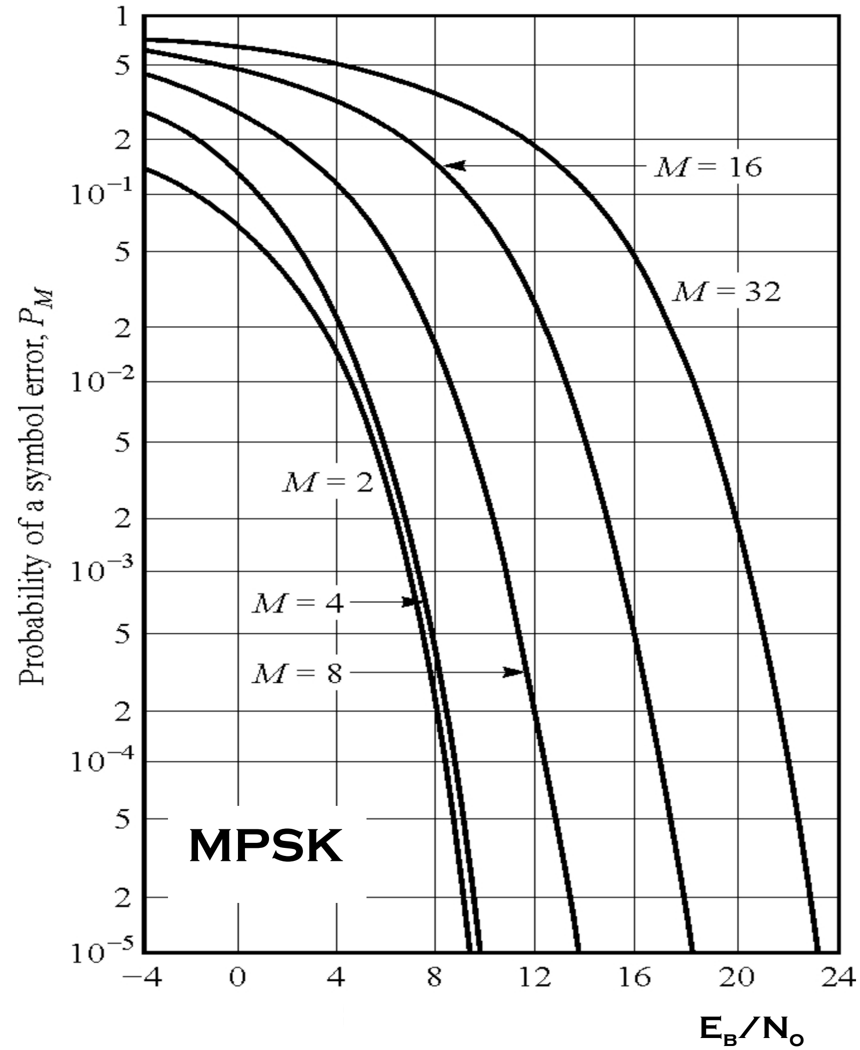


Note: Gray coding cannot be used for orthogonal FSK

PROBABILITY OF SYMBOL ERROR

M-QAM, M-PSK: BW-efficient but not power-efficient
For M>8, M-QAM outperforms M-PSK





EXAMPLE OF BPSK PERFORMANCE

AWGN CHANNEL:

$r[k] = x[k] + w[k]$ where $x[m] = \pm a$ with prob. of 1/2,

$E_b = a^2 T_b$ and $w[m]$: Gaussian $(0, N_o/2)$

$$P_e = Q\left(\sqrt{2E_b/N_o}\right) \approx 0.5e^{-E_b/N_o}$$

- Error probability decays exponentially with SNR

RAYLEIGH FLAT-FADING CHANNEL:

$r[k] = |h[k]|\, x[k] + w[k]$

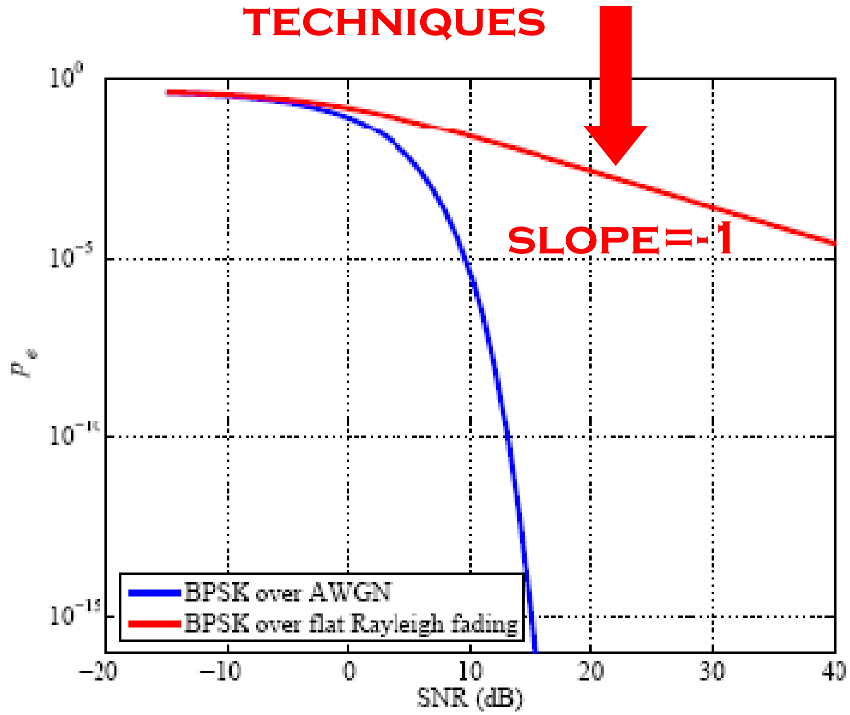
$|h[k]|$: Rayleigh with $\sigma^2 = 1$,

$$P_{s\|h[k]\|} = Q\left(|h[k]|\sqrt{2E_b/N_o}\right)$$

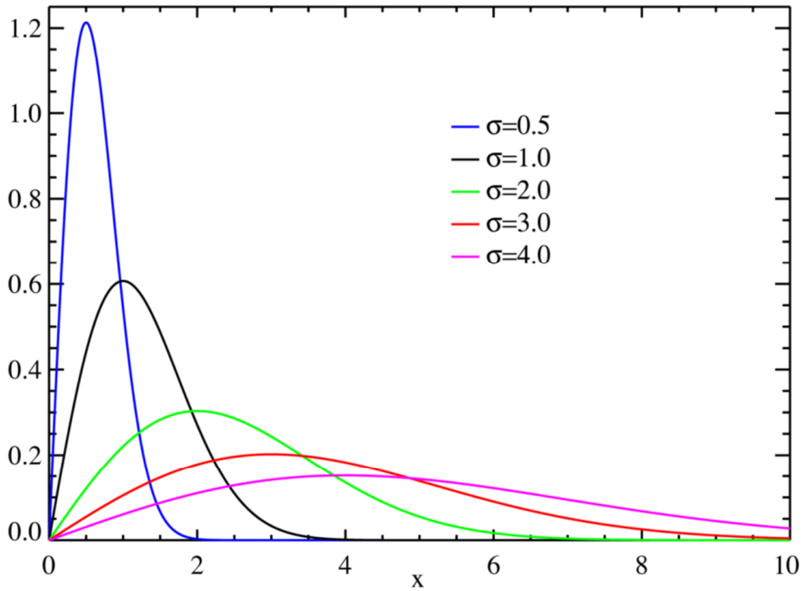
$$\overline{P}_s = \frac{1}{2}\left(1 - \sqrt{\frac{[E_b/N_o]}{1+[E_b/N_o]}}\right) \approx \frac{1}{4[E_b/N_o]}$$

- average error probability decays only inversely with SNR

NEEDS TO INCREASE THIS SLOPE BY DIVERSITY TECHNIQUES



RAYLEIGH, CHI, EXP, CHI-SQUARE



$Z = X_I + jX_Q : CN(0, \sigma^2)$,

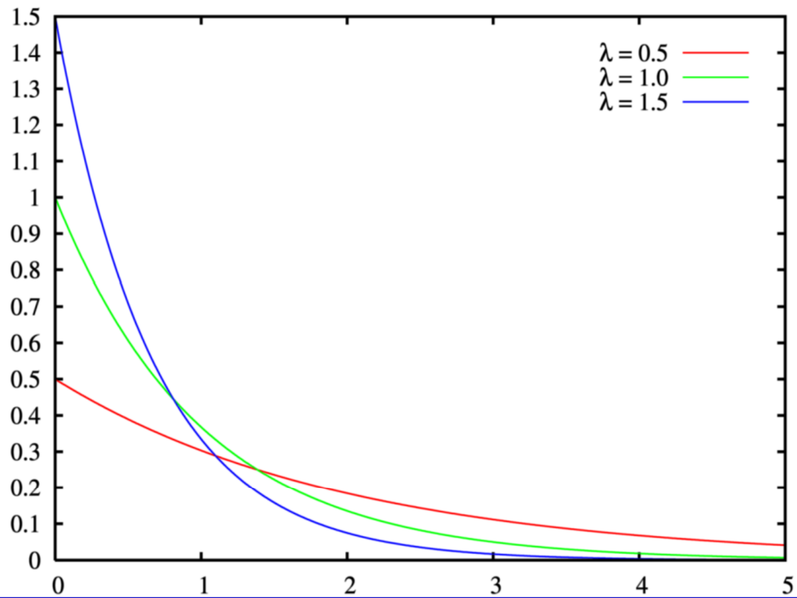
X_I, X_Q: $i.i.d.$ Gaussian, $N(0, \sigma^2)$

$X = \sqrt{X_I^2 + X_Q^2}$, $\quad X$: Rayleigh(σ^2)

X: also chi χ_2 with 2 degrees of freedom

$p_X(x) = \left[x / \sigma^2 \right] e^{-x^2/2\sigma^2}$,

$\overline{X} = \sigma\sqrt{\pi/2}$, var : $\sigma_X^2 = \sigma^2 \left[2 - (\pi/2) \right]$



$Y = X_I^2 + X_Q^2$, $\quad Y$: Exponential (λ)

Y: also chi-square χ_2^2

$p_Y(y) = \left[2\sigma^2 \right]^{-1} e^{-y/2\sigma^2}, \lambda = \left[2\sigma^2 \right]^{-1}$

$\overline{Y} = 2\sigma^2$, var : $\sigma_Y^2 = 4\sigma^4$

RICEAN

X_I, X_Q: *independent* Gaussian with same variance, $N(\mu_i, \sigma^2), i = I, Q$

$$X = \sqrt{X_I^2 + X_Q^2} : \text{Ricean } (\sigma^2),$$

$$p_X(x) = \left[x/\sigma^2 \right] I_0(sx/\sigma^2) e^{-\left(x^2+s^2\right)/2\sigma^2},$$

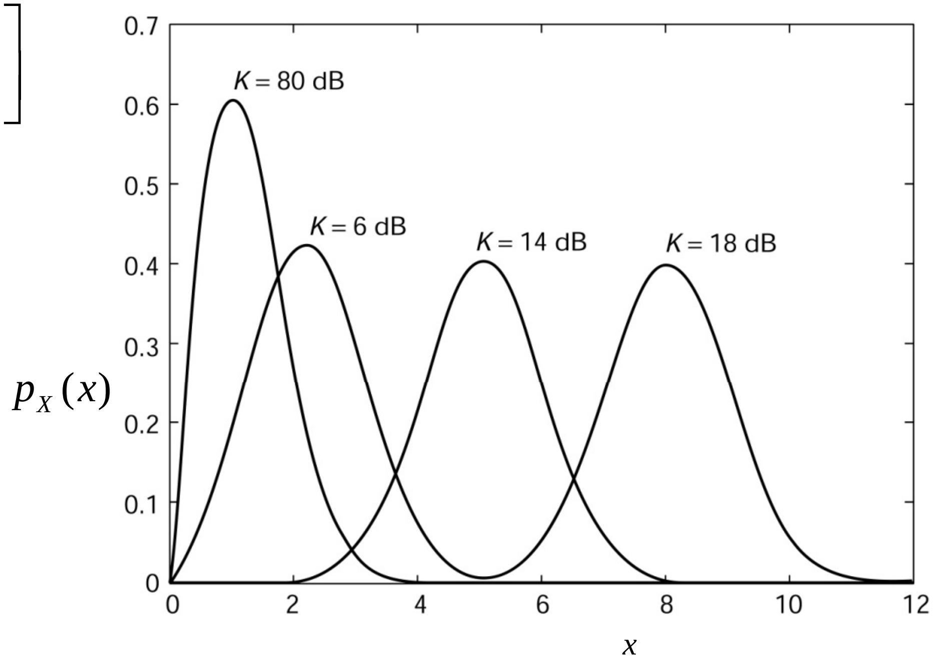
$$s = \sqrt{\mu_I^2 + \mu_Q^2}, \kappa = s^2/2\sigma^2, E\{X^2\} = 2\sigma^2 + s^2,$$

$$E\{X\} = e^{-\kappa/2} \sqrt{\frac{\pi\sigma^2}{2}} \left[(1+\kappa) I_0(\frac{\kappa}{2}) + \kappa + I_1(\frac{\kappa}{2}) \right]$$

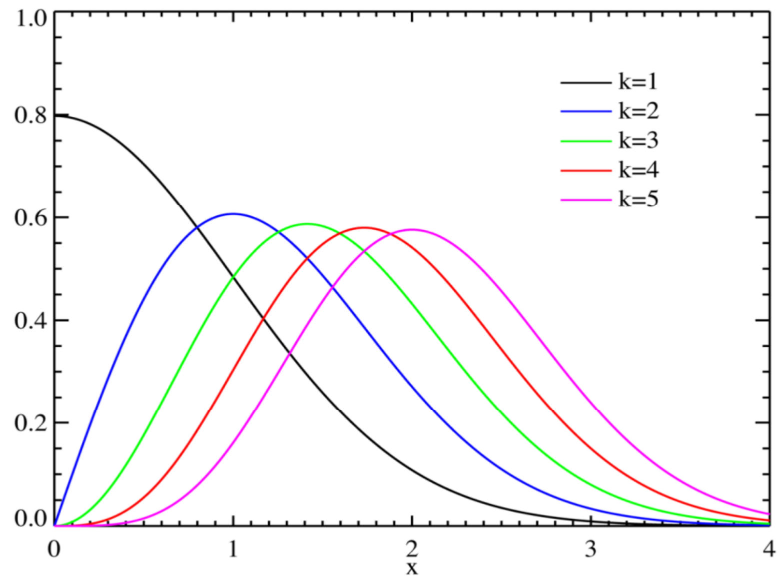
Bessel function of the 1st kind and order *a*:

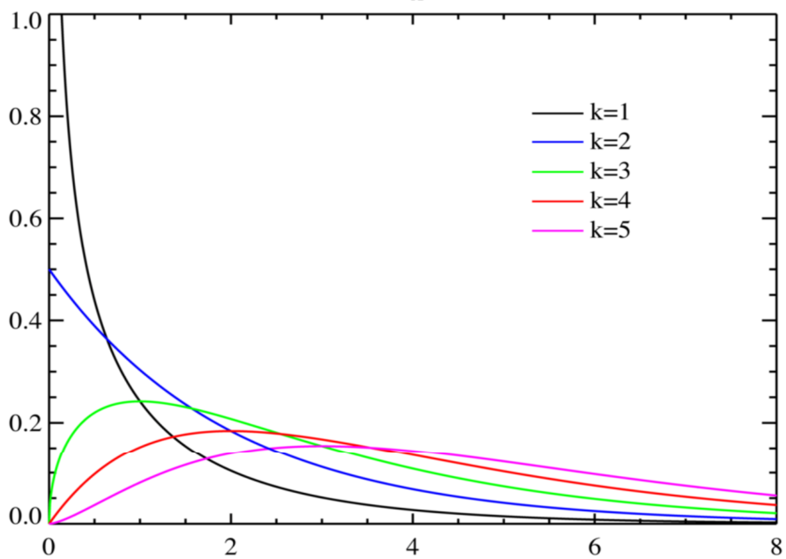
$$I_a(y) = \sum_{k=0}^{\infty} (y/2)^{a+2k} / [\Gamma(a+k+1)k!]$$

$$\rightarrow I_0(y) = \sum_{k=0}^{\infty} \left[y^k / (2^k k!) \right]^2$$



Gaussian, Chi, Chi-square





$X_i,\ i=1,2,\ldots,k$: independent *Gaussian* : $N(\mu_i,\sigma_i^2)$

$$X=\sqrt{\sum_{i=1}^{k}\left[\frac{X_i-\mu_i}{\sigma_i}\right]^2} : \chi_K$$

$$p_X(x)=\left[2^{k/2-1}\Gamma(k/2)\right]^{-1}x^{k-1}e^{-x^2/2},$$

$$E\left\{X^n\right\}=2^{n/2}\,\Gamma\left(\left[k+n\right]/2\right)\Big/\Gamma(k/2),$$

$$Y=\sum_{i=1}^{k}\left[\frac{X_i-\mu_i}{\sigma_i}\right]^2 :\ \chi_K^2$$

$$p_Y(y)=\left[2^{k/2}\Gamma(k/2)\right]^{-1}y^{k/2-1}e^{-y/2},$$

$$\overline{Y}=k,\ \text{var}:\sigma_Y^2=2k$$

Gaussian, Chi, Chi-square

$X_i,\ i=1,2,...,K$: i.i.d. $zero-mean$ Gaussian: $N(0,\sigma^2)$

$$X=\sqrt{\sum_{i=1}^{K}X_i^2}: \text{chi } \chi_K \text{ with } K \text{ degrees of freedom}$$

$$p_X(x)=\left[2^{K/2-1}\sigma^K\Gamma(K/2)\right]^{-1}x^{K-1}e^{-x^2/2\sigma^2}, E\left\{X^n\right\}=\left[2\sigma^2\right]^{n/2}\Gamma\left(\left[K+n\right]/2\right)\Big/\Gamma(K/2),$$

case K=2: Rayleigh, $p_X(x)=\sigma^{-2}xe^{-x^2/2\sigma^2}, E\left\{X\right\}=\sqrt{\pi\sigma^2/2}, E\left\{X^2\right\}=\left[2\sigma^2\right]$

$$Y=\sum_{i=1}^{K}X_i^2: \text{chi-square } \chi_K^2 \text{ with } K \text{ degrees of freedom}$$

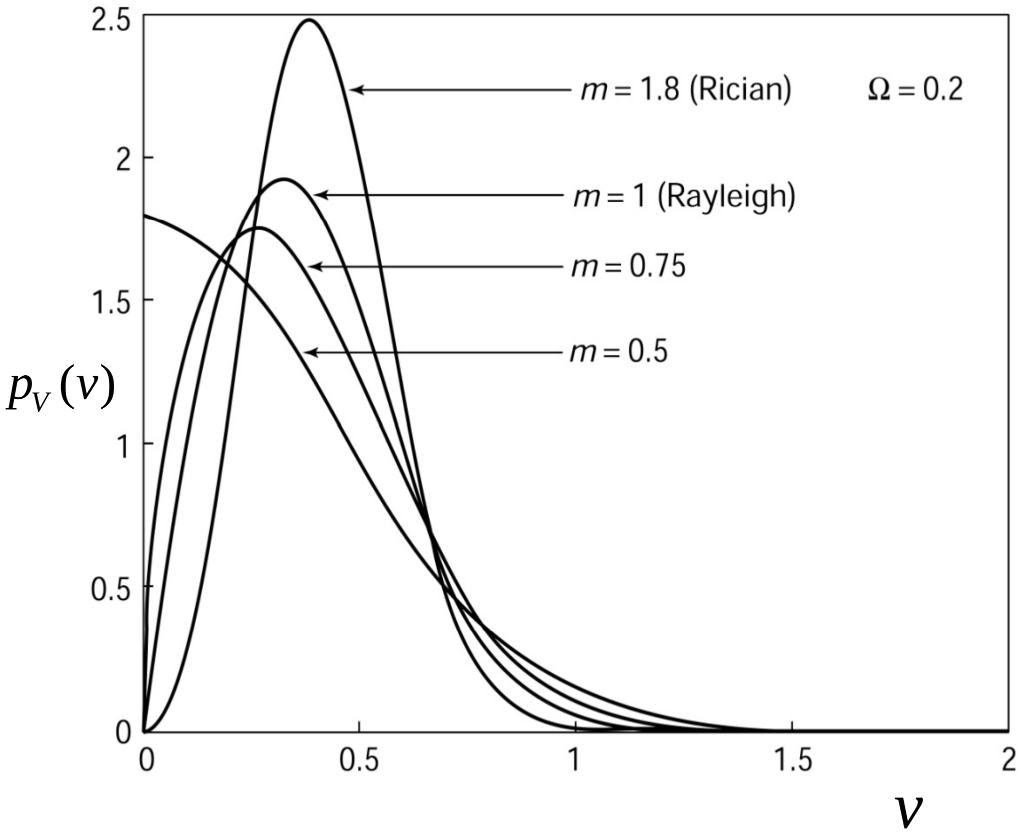
$$p_Y(y)=\left[2^{K/2}\sigma^K\Gamma(K/2)\right]^{-1}y^{K/2-1}e^{-y/2\sigma^2}, \overline{Y}=K\sigma^2, \text{var}: \sigma_Y^2=2K\sigma^4$$

case K=2: exponential, $p_Y(y)=\left[2\sigma^2\right]^{-1}e^{-y/2\sigma^2}, \overline{Y}=2\sigma^2, \text{var}: \sigma_Y^2=4\sigma^4$

$$\Gamma(u)=\int_0^\infty t^{u-1}e^{-t}dt, u>0, \Gamma(u+1)=u\Gamma(u), \Gamma(1/2)=\sqrt{\pi},$$

$$\Gamma(2)=\Gamma(1)=1, \quad \Gamma(n)=(n-1)! \text{ for } n: \text{integer} >1$$

Nakagami m-distribution:



V: Nakagami m-distributed:

$$p_V(v) = \left[2\left(m/\Omega\right)^m / \Gamma(m)\right] v^{2m-1} e^{-mv^2/\Omega}$$

$$\Omega = E\{V^2\}, E\{[V^2 - \Omega]^2\} = \Omega^2 / m$$

$m \geq 0.5$: fading figure,ratio of moments.

$$E\{V\} = (\sqrt{\Omega / m})\Gamma(m + 0.5) / \Gamma(m)$$

$$\Gamma(u) = \int_0^\infty t^{u-1} e^{-t} dt, u > 0,$$

$$\Gamma(u+1) = u\Gamma(u), \Gamma(1/2) = \sqrt{\pi},$$

$$\Gamma(2) = \Gamma(1) = 1,$$

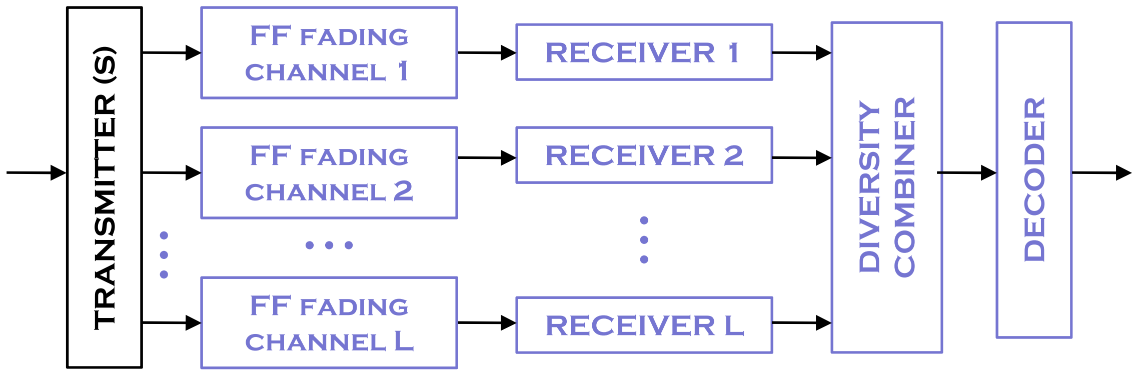
$\Gamma(n) = (n-1)!$ for n : integer > 1

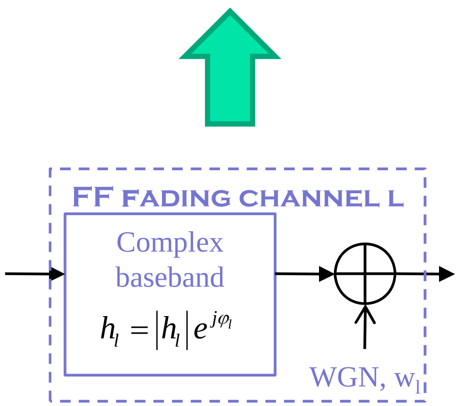
$m = 1 \rightarrow X$: Rayleigh, $p_X(x) = \sigma^{-2} x e^{-x^2/2\sigma^2}$

$$E\left\{X^2\right\} = \left[2\sigma^2\right], E\left\{X\right\} = \sqrt{\pi\sigma^2 / 2}$$

- pdf converges to a delta function for increasing *m*.
- matched empirical results for short wave ionospheric propagation.

Diversity Approach for frequency-flat fading channels





- **Multiple independent paths (or channels) unlikely to fade simultaneously**
- ⇒ Diversity techniques:
  - Send the same signals over independent fading paths obtained by diversity in time, space, frequency, …
    - ⇒ reduced possibility of all paths in deep fading simultaneously
  - Combine paths to mitigate fading effects

DIVERSITY SCHEMES

Time diversity (1 Tx+ 1 Rx):

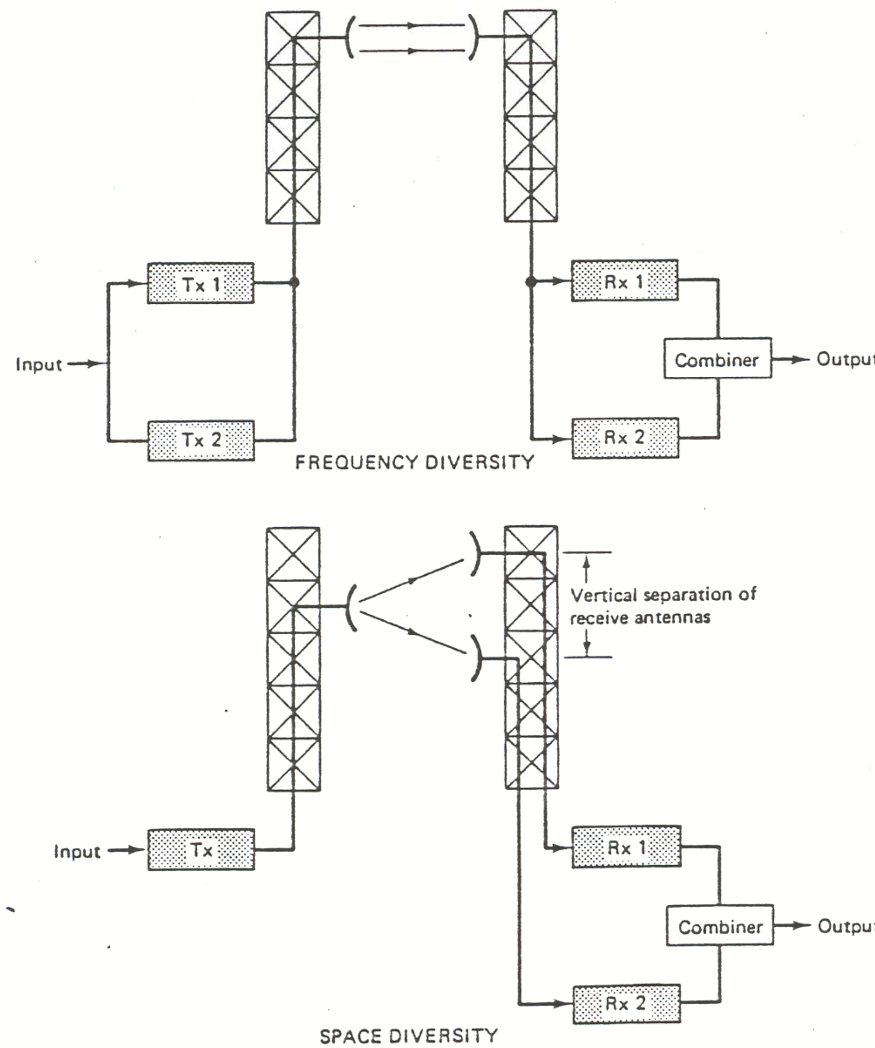
- multiple transmission of the same information over different time slots
- time separation > channel coherence time

Frequency diversity (L Tx + L Rx):

- multiple transmission of the same information over different frequency slots
- frequency separation > channel coherence bandwidth

Space diversity (1-L Tx + 1-L Rx):

- transmission to multiple antennas
- Sufficient antenna separation to achieve uncorrelated channel gains, e.g., about half wavelength, $\lambda/2$, for a Rayleigh fading channel.

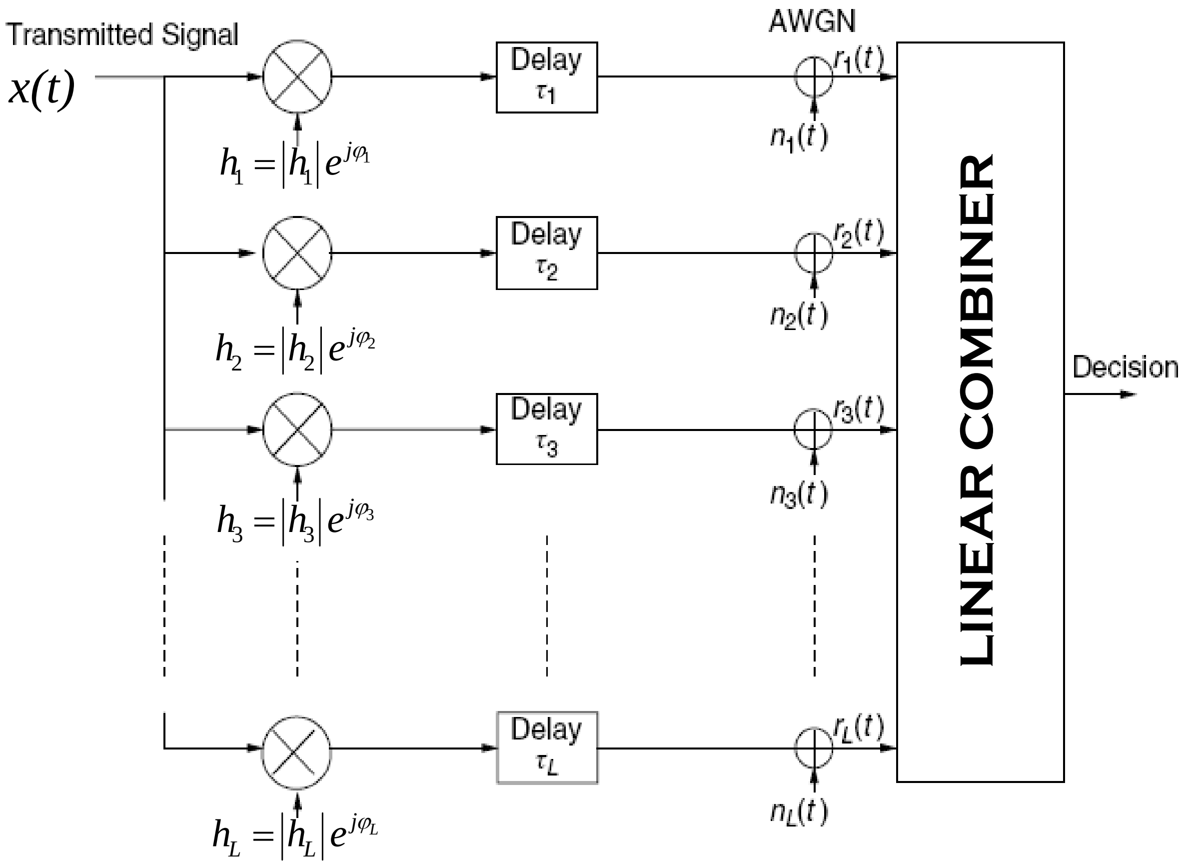


Diversity Techniques at Receiver

- **Transmitter** sends the same signals over L independent fading paths obtained by diversity in time, space, frequency, …: **Simplest Coding: Repetition**

$$\mathbf{y}(k)=\mathbf{h}x(k)+\mathbf{n}(k),\ \mathbf{y}(k)=\left[y_l(k), l=1,..,L\right]^T,\ \mathbf{h}(k)=\left[h_l(k), l=1,..,L\right]^T$$

$$\mathbf{n}(k)=\left[n_l(k), l=1,..,L\right]^T\ n_l(k): i.i.d.\ AWGN$$



Receiver combines paths to mitigate fading effects

- classic vector detection in white Gaussian noise
- Diversity gain is indicated by changes in BER slope

Combining Techniques:

- **Maximal Ratio Combining (MRC):** all paths cophased and summed with optimal weights to maximize combiner output SNR
- **Equal Gain Combining (EGC):** All paths co-phased and summed with equal weights
- **Selection Combining (SC):** select the fading path with the highest gain

Selection combining over Rayleigh fading

received vector: $\mathbf{r}(k) = \mathbf{h}x(k) + \mathbf{n}(k)$, $\mathbf{h} = [h_1, h_2, ..., h_L]^T$

Rayleigh channel: $h_l = |h_l| e^{j\varphi_l}$, $l = 1, 2, .., L$: i.i.d., $|h_l|$:Rayleigh

$Y = |h_l|^2 > 0$: exponential, $p_Y(y) = \left[2\sigma^2\right]^{-1} e^{-y/2\sigma^2}$, $\overline{Y} = 2\sigma^2$, var : $\sigma_Y^2 = 4\sigma^4$

select the max $|h_*|$ and coherently demodulate:

$$\tilde{r}(k) = |h_*| x(k) + n_*(k),\ |h_*| = \max\left\{|h_l|, l = 1, 2, .., L\right\}$$

$$\text{cdf:} \Pr\left\{|h_*|^2 \le y\right\} = \Pr\left\{\bigcap_{l=1}^{L} |h_l|^2 \le y\right\} = \left[\int_0^y \left[2\sigma^2\right]^{-1} e^{-x^2/2\sigma^2} dx\right]^L$$

$$p_{|h_*|^2}(y) = \frac{d \Pr\left\{|h_*|^2 \le y\right\}}{dy} = \frac{L}{2\sigma^2} e^{-y/2\sigma^2} \left[1 - e^{-y/2\sigma^2}\right]^{L-1}, y \ge 0$$

$SNR_{SC} = |h_*|^2 [E_s / N_o]$ as compared to non-diversity case: $SNR = |h_l|^2 [E_s / N_o]$

BPSK: $P_{s||h_*|} = Q\left(\sqrt{2|h_*|^2 E_b / N_o}\right)$

$$\to \overline{P}_s = \left[L/(2\sigma^2)\right] \int_0^\infty Q\left(\sqrt{2yE_b / N_o}\right) e^{-y/2\sigma^2} \left[1 - e^{-y/2\sigma^2}\right]^{L-1} dy$$

solved by numerical integration

Selection Combining (SC) Diversity in Rayleigh channels

Rayleigh channel: $h_l = |h_l| e^{j\varphi_l}, l = 1, 2, .., L : i.i.d., |h_l|$:Rayleigh

$Y = |h_l|^2 > 0$: exponential, $p_Y(y) = \left[2\sigma^2\right]^{-1} e^{-y/2\sigma^2}, \bar{Y} = 2\sigma^2, \text{var} : \sigma_Y^2 = 4\sigma^4$

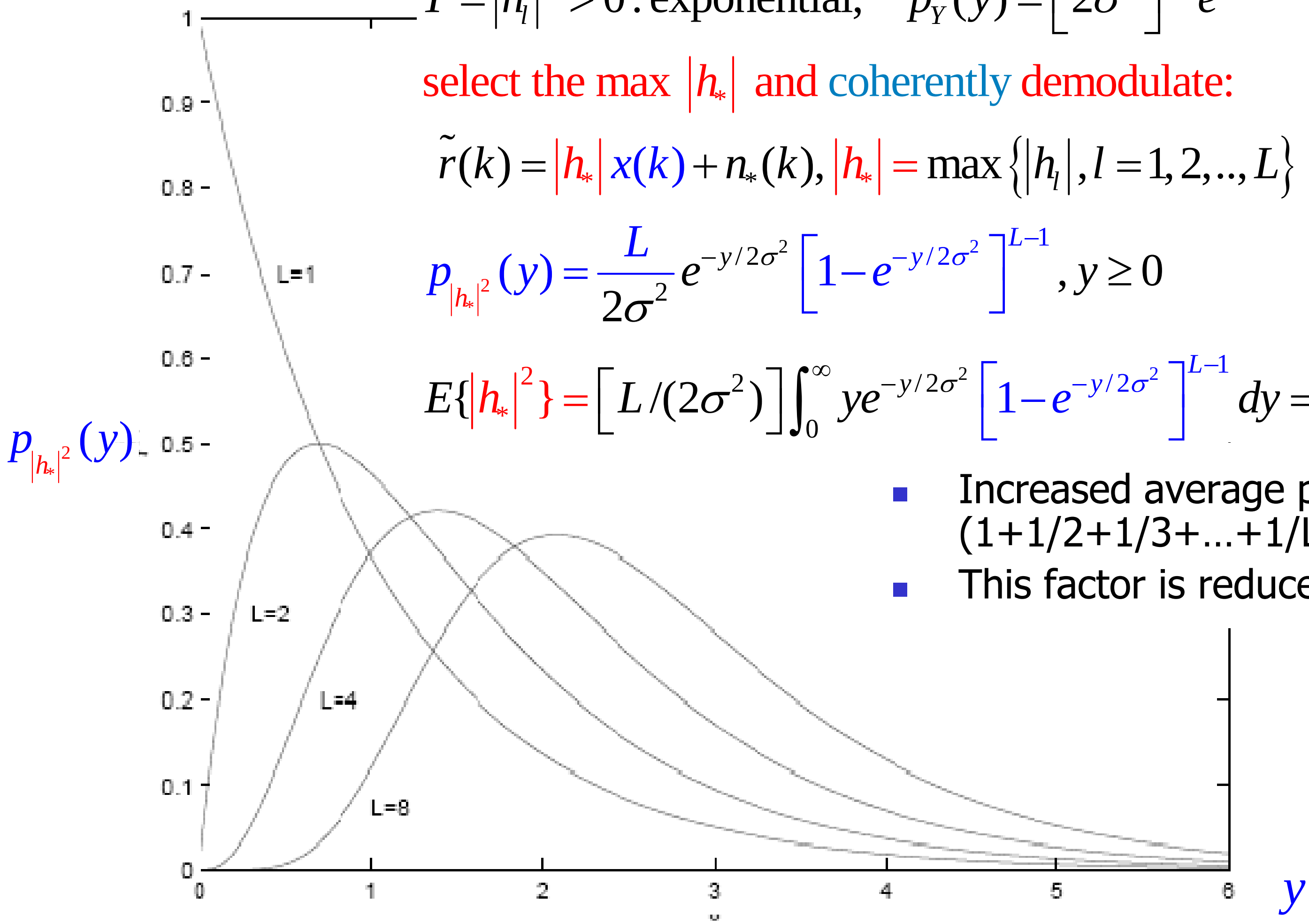
select the max $|h_*|$ and coherently demodulate:

$$\tilde{r}(k) = |h_*| x(k) + n_*(k), |h_*| = \max\{|h_l|, l = 1, 2, .., L\}$$

$$p_{|h_*|^2}(y) = \frac{L}{2\sigma^2} e^{-y/2\sigma^2} \left[1 - e^{-y/2\sigma^2}\right]^{L-1}, y \geq 0$$

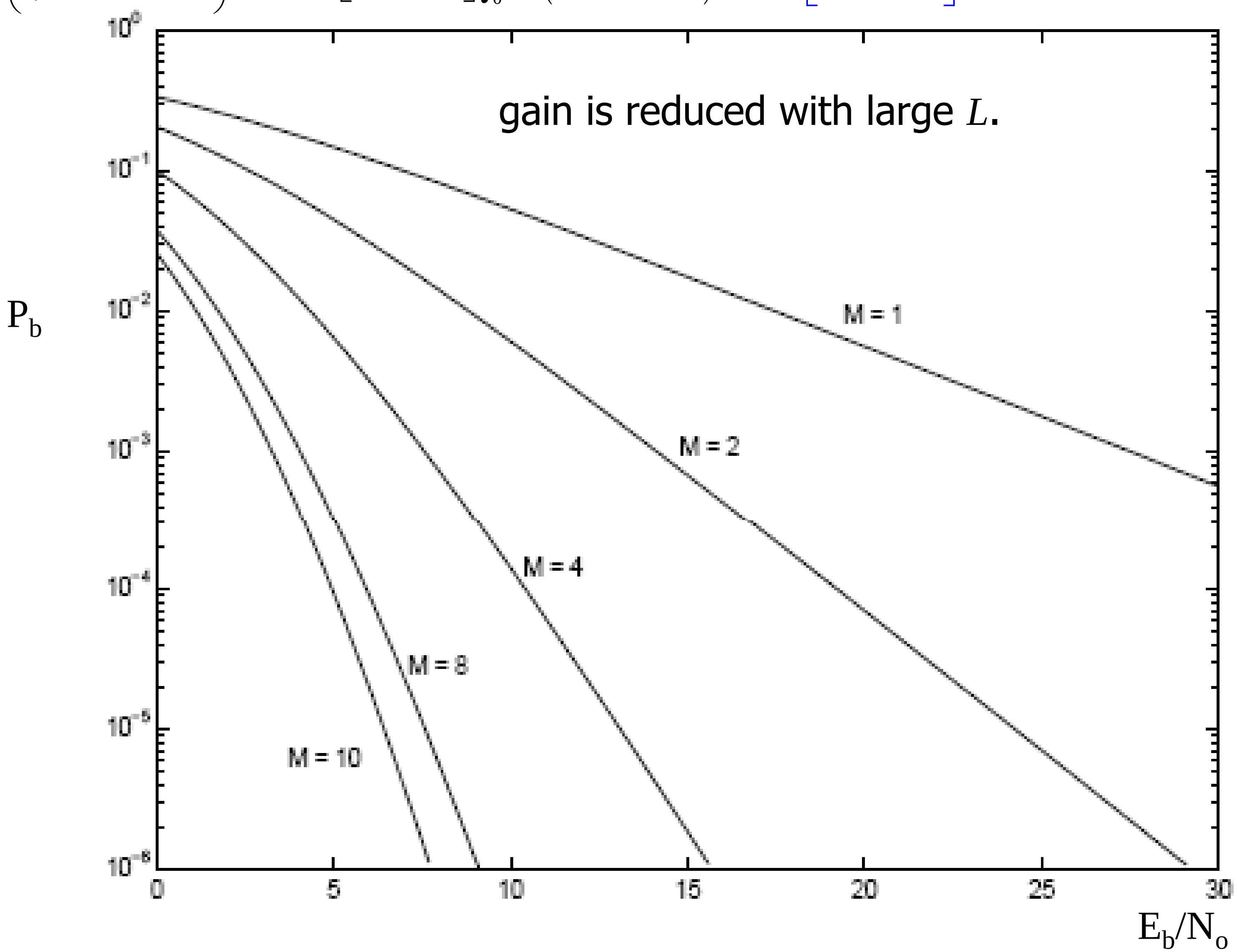
$$E\{|h_*|^2\} = \left[L/(2\sigma^2)\right] \int_0^\infty y e^{-y/2\sigma^2} \left[1 - e^{-y/2\sigma^2}\right]^{L-1} dy = (2\sigma^2) \sum_{l=1}^{L} l^{-1} \geq (2\sigma^2)$$

- Increased average path gain by a factor (1+1/2+1/3+…+1/L)
- This factor is reduced with large L.



Selection combining: performance of BPSK over Rayleigh fading

BPSK: $P_{s\|h_*\|} = Q\left(\sqrt{2|h_*|^2 E_b/N_o}\right) \rightarrow \overline{P}_s = \left[L/(2\sigma^2)\right]\int_0^\infty Q\left(\sqrt{2yE_b/N_o}\right)e^{-y/2\sigma^2}\left[1-e^{-y/2\sigma^2}\right]^{L-1}dy$



EGC & its Performance in Rayleigh Channels

received vector: $\mathbf{r}(k) = \mathbf{h}x(k) + \mathbf{n}(k)$,

Rayleigh channel: $h_l = |h_l| e^{j\varphi_l}$, $l = 1, 2, .., L$: $i.i.d.$, $|h_l|$:Rayleigh

$Y = |h_l|^2 > 0$: exponential, $p_Y(y) = \left[2\sigma^2\right]^{-1} e^{-y/2\sigma^2}$, $\overline{Y} = 2\sigma^2$, var : $\sigma_Y^2 = 4\sigma^4$

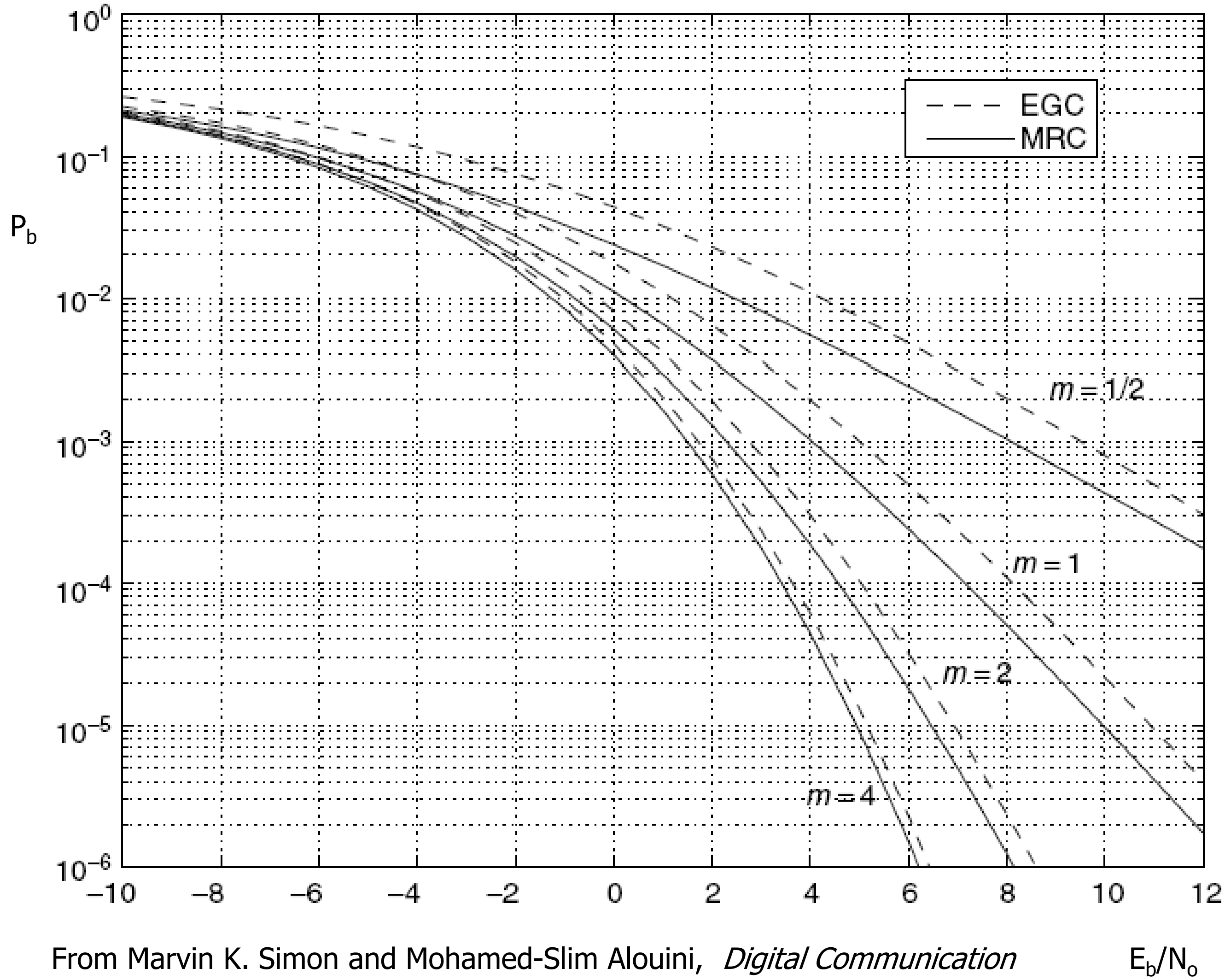
Coherently demodulate and combine with *equal* weights: $\mathbf{\Phi_h} = [e^{-j\varphi_1}, e^{-j\varphi_2}, 2, .., e^{-j\varphi_L}]$

$$\tilde{r}(k) = \mathbf{\Phi_h}\mathbf{r}(k) = h_{sum}x(k) + w(k),\ h_{sum} = \left[\sum_{l=1}^{L} |h_l|\right], w(k) = \mathbf{\Phi_h}\mathbf{n}(k) : Gaussian(0, LN_o/2)$$

$SNR_{EGC} = \left[h_{sum}^2 / L\right]\left[E_s / N_o\right]$ as compared to non-diversity case: $SNR = |h_l|^2 \left[E_s / N_o\right]$

BPSK: $P_{s|h_{sum}} = Q\left(\sqrt{2h_{sum}^2 E_b / N_o}\right)$

Average BER of BPSK over Nakagami-m fading channels with MRC and EGC (L = 4).



From Marvin K. Simon and Mohamed-Slim Alouini, *Digital Communication over Fading Channels, 2nd Edition,* John Wiley & Sons, 2005

MRC & its Performance in Rayleigh Channels

received vector: $\mathbf{r}(k) = \mathbf{h}x(k) + \mathbf{n}(k)$, $\mathbf{h} = [h_1, h_2, ..., h_L]^T$

Rayleigh channel: $h_l = |h_l| e^{j\varphi_l}$, $l = 1, 2, .., L : i.i.d.$, $|h_l|$:Rayleigh

$Y = |h_l|^2 > 0$: exponential, $\quad p_Y(y) = \left[2\sigma^2\right]^{-1} e^{-y/2\sigma^2}$

$\overline{Y} = 2\sigma^2$, var : $\sigma_Y^2 = 4\sigma^4$

select the max $|h_*|$ and coherently demodulate and combine with optimum weights, $\mathbf{h}^H$:

matched filter: $\tilde{r}(k) = \mathbf{h}^H \mathbf{r}(k) = \|\mathbf{h}\|^2 x(k) + w(k)$, $w(k) = \mathbf{h}^H \mathbf{n}(k) : Gaussian(0, \|\mathbf{h}\|^2 N_o / 2)$

MAX OUTPUT SNR

instantaneous $SNR_{EGC} = \left[\|\mathbf{h}\|^2\right]\left[E_s / N_o\right]$ as compared to non-diversity case: $SNR = |h_l|^2 \left[E_s / N_o\right]$

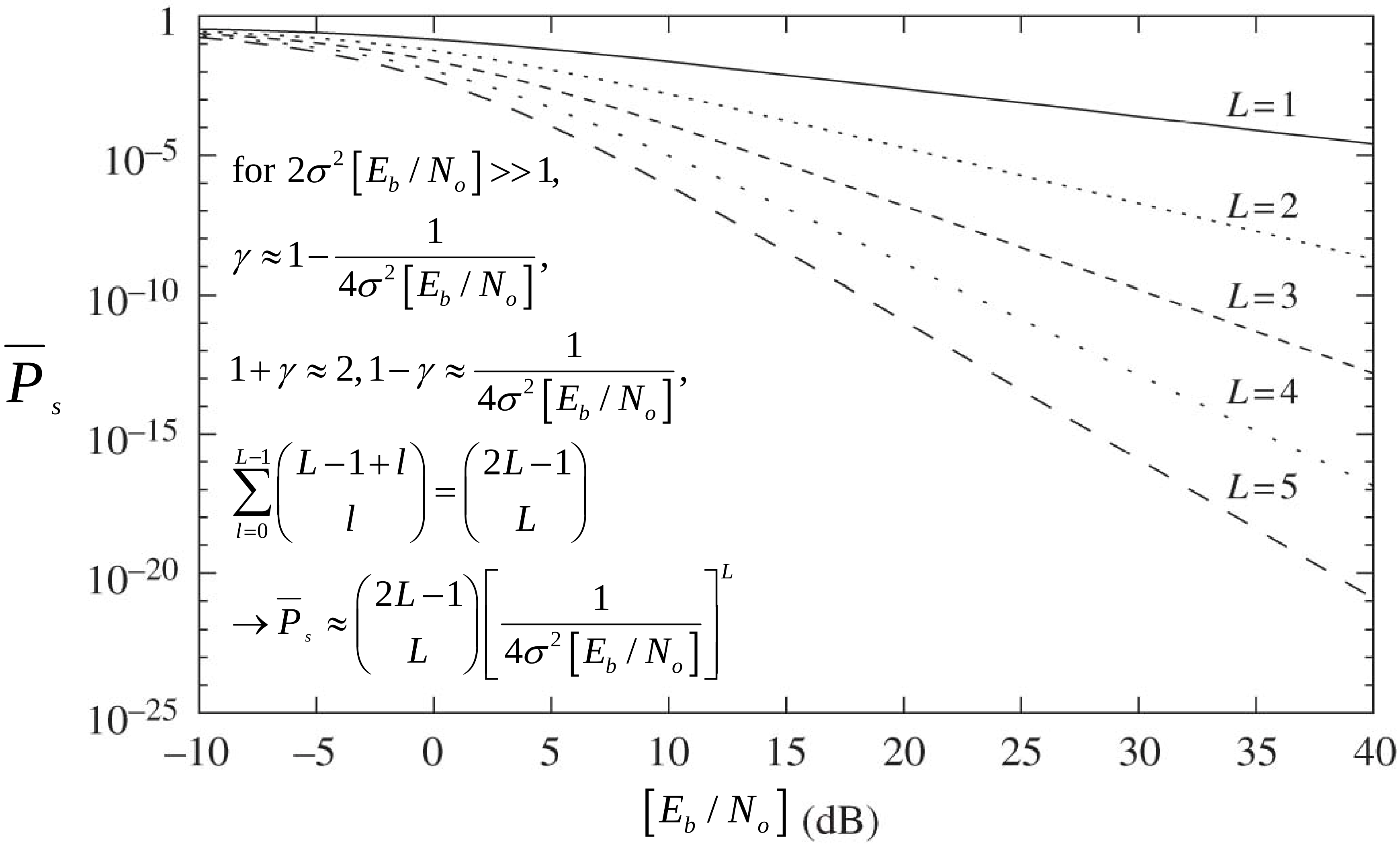
$Y = \|\mathbf{h}\|^2$, $p_Y(y) = \left[2^L \sigma^{2L} (L-1)!\right]^{-1} y^{L-1} e^{-y/2\sigma^2}$, $\overline{Y} = 2L\sigma^2$, var : $\sigma_Y^2 = 4L\sigma^4$

BPSK: $P_{s|\|h\|} = Q\left(\sqrt{2\|\mathbf{h}\|^2 E_b / N_o}\right)$

$$\rightarrow \overline{P}_s = \left[2^L \sigma^{2L} (L-1)!\right]^{-1} \int_0^\infty Q\left(\sqrt{2yE_b / N_o}\right) y^{L-1} e^{-y/2\sigma^2} dy$$

$$\overline{P}_s = \left[\frac{1-\gamma}{2}\right]^L \sum_{l=0}^{L-1} \binom{L-1+l}{l} \left[\frac{1+\gamma}{2}\right]^l, \gamma = \sqrt{\frac{2\sigma^2 \left[E_b / N_o\right]}{1 + 2\sigma^2 \left[E_b / N_o\right]}}$$

Performance of BPSK using MRC in Rayleigh Channels



for $2\sigma^2[E_b/N_o] >> 1$,

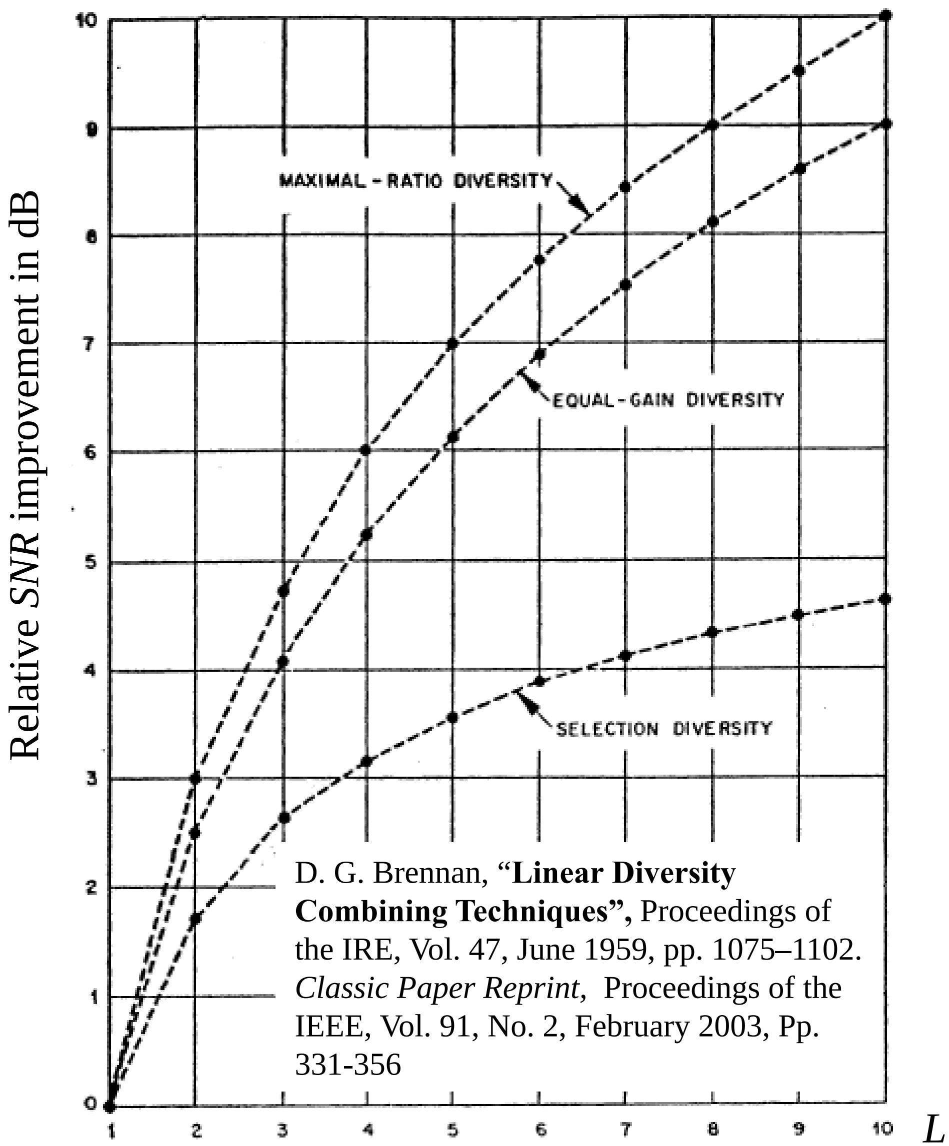
$$\gamma \approx 1 - \frac{1}{4\sigma^2[E_b/N_o]},$$

$$1+\gamma \approx 2, 1-\gamma \approx \frac{1}{4\sigma^2[E_b/N_o]},$$

$$\sum_{l=0}^{L-1}\binom{L-1+l}{l} = \binom{2L-1}{L}$$

$$\rightarrow \overline{P}_s \approx \binom{2L-1}{L}\left[\frac{1}{4\sigma^2[E_b/N_o]}\right]^L$$

Performance & complexity



D. G. Brennan, "**Linear Diversity Combining Techniques**", Proceedings of the IRE, Vol. 47, June 1959, pp. 1075–1102. *Classic Paper Reprint*, Proceedings of the IEEE, Vol. 91, No. 2, February 2003, Pp. 331-356

Selection combining (SC):

- The receiver monitors the SNR of the received signal from each diversity branch, and, selects *only* the Rx signal corresponding to the highest SNR *for detection;*
- Simple but low performance.

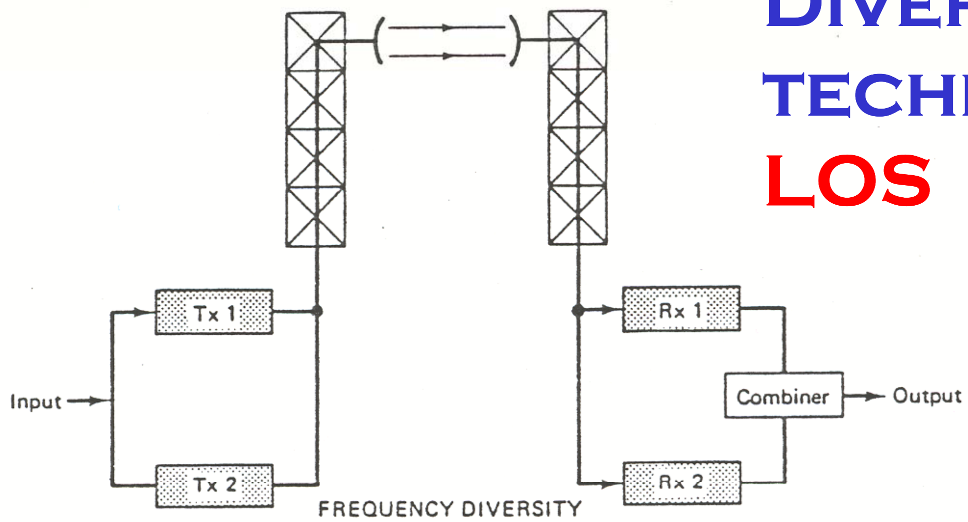
Equal gain combining (EGC):

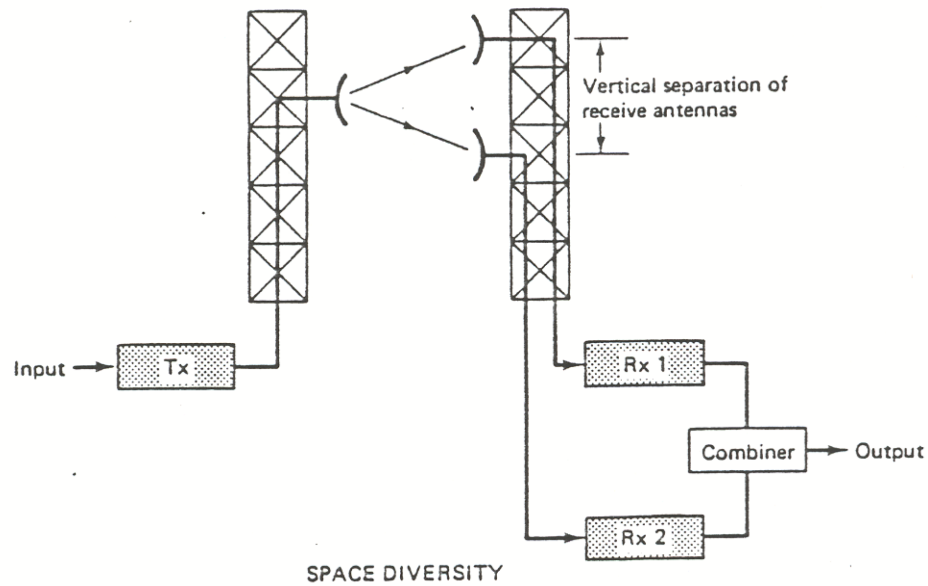
- The received signals from the *L* diversity branches are *coherently* combined with *equal* weights
- The receiver does not need the information of $||\mathbf{h}||$
- Performance is worse than that of MRC (about 1 dB), but much better than SC for large *L*

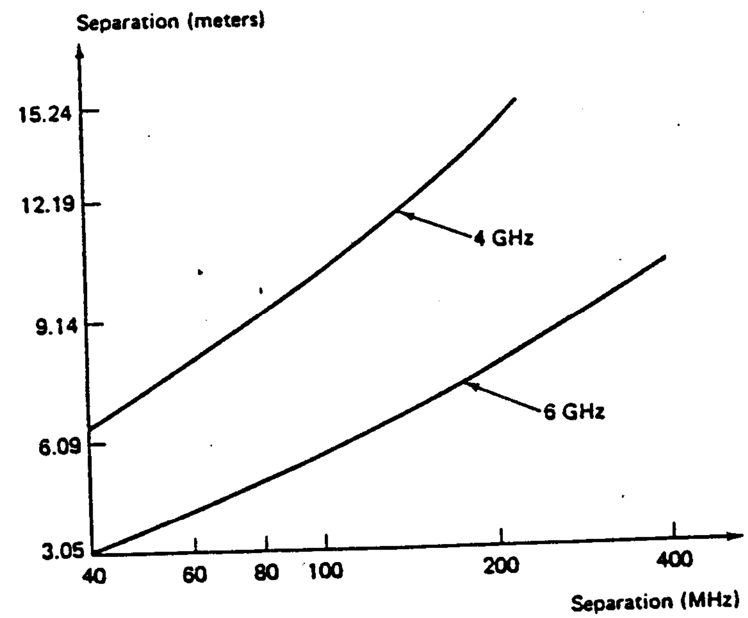
Maximal ratio combining (MRC):

- The received signals from the *L* diversity branches are *coherently* combined with *optimum* weights
- The receiver must know $\mathbf{h}$
- Optimum performance

Diversity techniques in LOS Systems







Separation in 1-for-1 space and frequency diversity systems providing equal availability time-improvement factors. Equal size antennas have been assumed. (Redrawn with permission from the *Bell System Technical Journal*, copyright 1975, The American Telephone and Telegraph Company)

Diversity improvement in LOS Systems

improvement in dB for fading>20dB

SPACE DIVERSITY (A. Vigants): $I_{SD}=(1.2E-3)(S^2f/d) .10^{-\Delta G/20}\ 10^{F/10}$

S = vertical separation of equal size antennas in meters
f = frequency in GHz
ΔG = effective gain difference between the two antennas, due either to different sizes
or different waveguide run losses, referenced to the basic path reliability calculations
F = median fade depth in decibels
d = path length in kilometers
For the specular reflection or overwater problem, one alternative is to select the antenna separation on the receive terminal for $S = 246d/fh_t$
S = antenna separation in meters
d = path length in kilometers
f = frequency in GHz
h_t = heights of the transmitter antenna above the water in meters

FREQUENCY DIVERSITY (W.T. Barnett): $I_{FD} = 10\log [\ K\ 10^{F/10}\ \Delta f/f]$

Δf = difference between two RF carriers, in GHz
f = RF carrier in GHz
$K=(1.1)10^{-a}$, $a=\log(f^{0.8})$, empirically derived frequency-dependent constant
F = multipath fade depth in dB, normally set equal to equipment fade margin

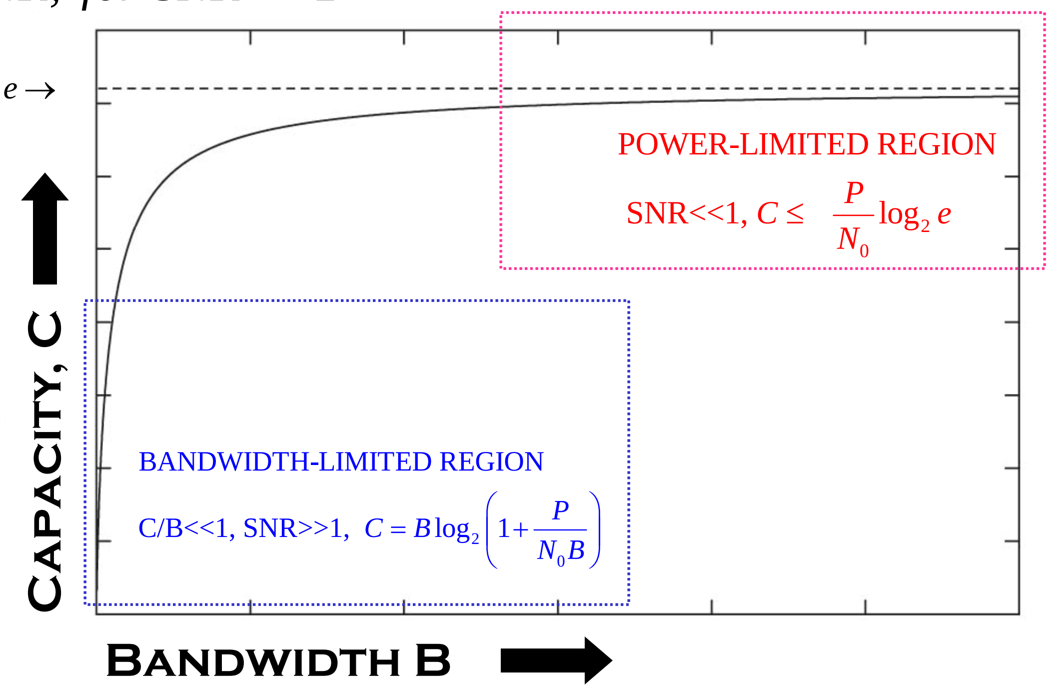
Capacity of AWGN Channel

$$y(k)=x(k)+n(k),\ n(k): AWGN$$

Capacity: $$\left(\frac{C}{B}\right)=\log_2\left(1+\frac{E_b}{N_0}\left(\frac{C}{B}\right)\right)\ b/s/Hz,\ \frac{E_b}{N_0}\left(\frac{C}{B}\right)=\frac{P}{N_0 B}=SNR$$

$$\Rightarrow C = B\log_2\left(1+SNR\right)\approx\begin{cases}\frac{P}{N_0}\log_2 e,\ \ for\ SNR\approx 0\\ B\log_2 SNR,\ \ for\ SNR>>1\end{cases}$$

$C\le\frac{P}{N_0}\log_2 e\rightarrow$



CAPACITY IN

- POWER-LIMITED REGION:
  - linear in power,
  - insensitive to bandwidth
- BANDWIDTH-LIMITED REGION:
  - logarithmic in power,
  - approximately linear in bandwidth

Frequency-flat fading channel capacity:

- Theoretical limit on maximum error-free Tx rate a channel can support due to channel characteristics and not dependent on design techniques
- It depends on what is known about channel fading.
  - No knowledge: Worst-case channel capacity
  - Partial knowledge of fading statistics
  - Full knowledge of fading level at **receiver only**

$$C = \int_0^\infty B\log_2\left(1 + aSNR\right) p(a)da \le B\log_2(1 + \overline{a}SNR)$$ (using Jensen's inequality)

  - Full knowledge of fading level at **both transmitter and receiver**:
    - For fixed transmitted power, same as above
    - transmit power can be adapted to a for optimum results

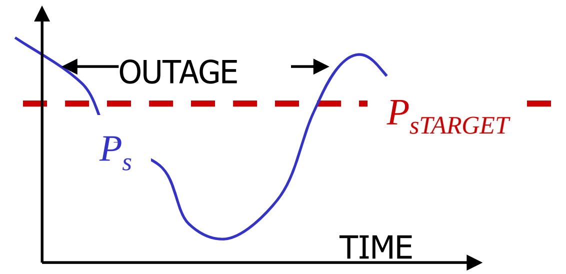
$$C = \max_{SNR(a) | E[SNR(a)] = \overline{SNR}} \int_0^\infty B\log_2\left(1 + aSNR(a)\right) p(a)da$$

Slow Flat-Fading Channels
Fading Known at Receiver

$y(k) = hx(k) + n(k),\ n(k): AWGN, \quad a = |h|^2$: power fading with pdf $p(a)$

instantaneous: $P_s(a) \approx A_M Q\left(\sqrt{B_M^2 (aSNR)}\right), C(a) = B\log_2(1 + aSNR)\ \ b/s,$

For **Slow Fading**: $T_{symbol} << T_{coherence}$



Performance outage probability:

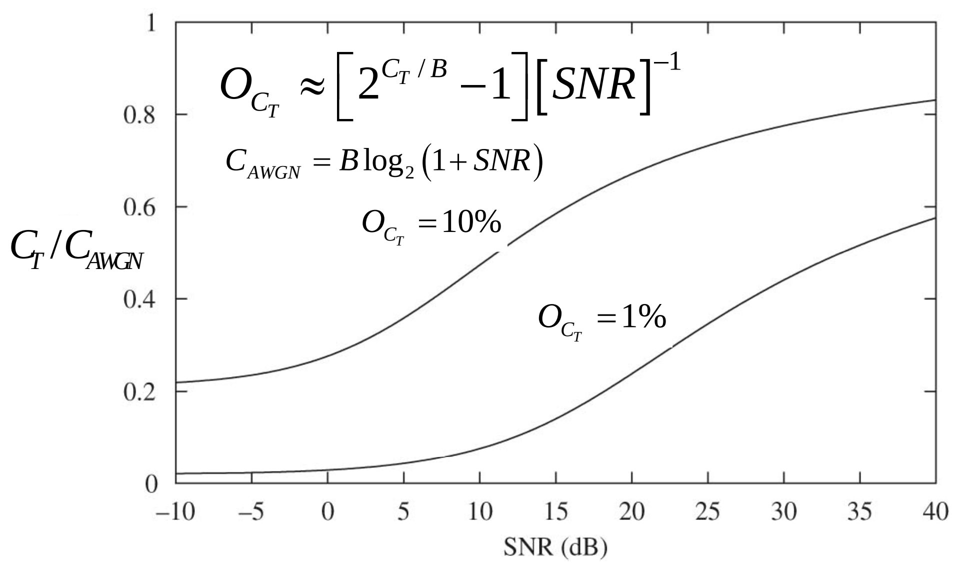
$$O_{P_T} = \Pr\{P_s(a) > P_{sTARGET}\} = \Pr\{a < a_{TARGET}\}$$

Capacity outage probability:

$$O_{C_T} = \Pr\{B\log_2(1 + aSNR) \le C_T\}$$

for $a^{1/2}$: Rayleigh-distributed,

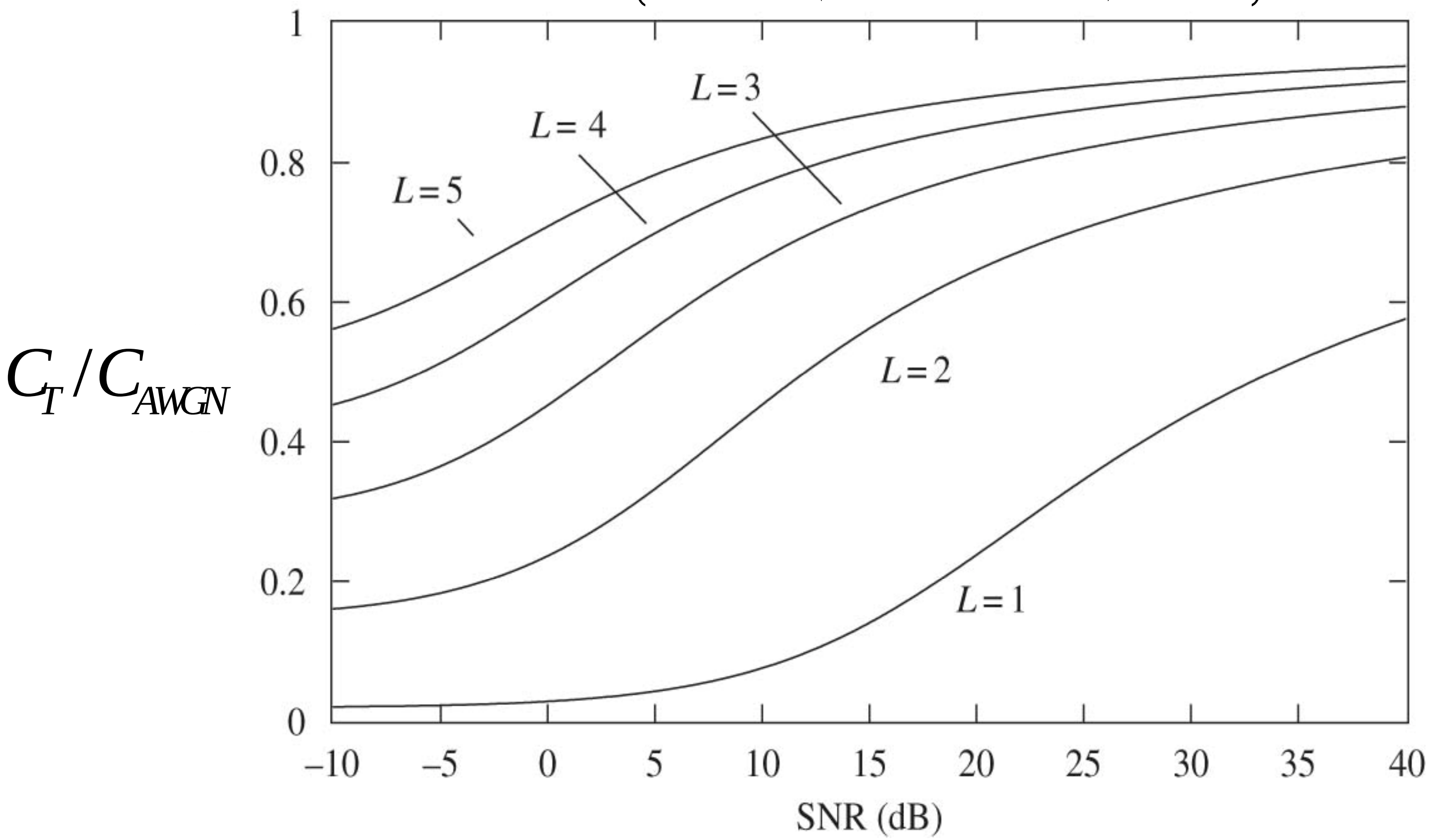
$$O_{C_T} \approx \left[2^{C_T/B} - 1\right][SNR]^{-1}$$



Receive Diversity: Capacity Outage Probability

Capacity outage probability:

$$O_{C_T} = \Pr\left\{B\log_2\left(1+\|\mathbf{h}\|^2 SNR\right) \le C_T\right\}$$



Fast Flat-Fading Channels
Fading Known at Receiver

$y(k) = hx(k) + n(k), n(k): AWGN, \quad a = |h|^2$: power fading with pdf $p(a)$

instantaneous: $P_s(a) \approx A_M Q\left(\sqrt{B_M^2 (aSNR)}\right), C(a) = B\log_2(1+aSNR)\ b/s,$

For **Fast Fading**, $T_{symbol} \approx T_{coherence}$ or $> T_{coherence}$,

- introduced random phase can remove correlation between symbol phases, and hence leads to an irreducible error floor for differential modulation/demodulation.
- For coherent demodulation, if fading is known by receiver only:

$$\text{Average (symbol) error probability: } \overline{P}_s = \int_0^{\infty} P_s(a)\,p(a)\,da$$

$$\text{ergodic:} C = E\{C(a)\} = B\int_0^{\infty} \log_2(1+aSNR)\,p(a)\,da \le B\log_2(1+SNR)$$

(using Jensen's inequality)

> i.e., If **Fading Known at Receiver Only**, at best, ergodic C approaches C_{AWGN}

Capacity of Flat-Fading Channels

$$y(k) = hx(k) + n(k),\ n(k): AWGN,\ a = |h|^2$$

$$\text{instantaneous:} C(a) = B\log_2\left(1 + aSNR\right)\ b/s,$$

$$\text{ergodic:} C = E\left\{C\left(a\right)\right\} = B\int_0^\infty \log_2\left(1 + aSNR\right)p(a)da \le B\log_2\left(1 + SNR\right)$$

(using Jensen's inequality)

- If **Fading Known at Receiver Only**, at best, approaches C_{AWGN}
- If **Fading also Known at Transmitter**,
  - using fixed transmitted power, SNR, same capacity as case of fading knowledge at receiver only,
  - using **adaptive** transmitted power, $A(a)SNR$, is it better?

$$\text{instantaneous: } C\left[A\left(a\right)\right] = \log_2\left[1 + aA(a)SNR\right]$$

$$\Rightarrow \text{ergodic:} C_A = \int_0^\infty \log_2\left[1 + aA(a)SNR\right]p(a)da$$

- **Can have a capacity greater than that of the AWGN channel, i.e., fading can provide more opportunities for performance enhancement in an opportunistic communication approach.**

Flat- Fading Also Known at Transmitter: using Adaptive Channel Inversion

- **Channel Inversion:** $A(a)=1/a$ to maintain a constant SNR_{ZO} for zero outage (e.g., same as perfect power control in CDMA) under average Tx power constraint :
  - Simplifies design (i.e., fixed rate at all channel states) but is power-inefficient since for very small a, $A(a)=1/a$ is very large.

$$A(a)=1/a \rightarrow C\left[A(a)\right]=\log_2\left[1+aA(a)SNR_{ZO}\right]=\log_2\left[1+SNR_{ZO}\right]$$

$$\text{but with Tx power constraint:} \int_0^\infty A(a)p(a)da=1 \rightarrow E\left\{A(a)\right\} = E\left\{1/a\right\}$$

$$\Rightarrow \text{ergodic, zero-outage:} C_{ZO}=\log_2\left[1+SNR_{ZO}\right] \text{ with } SNR_{ZO}=\frac{SNR}{E\left\{1/a\right\}}$$

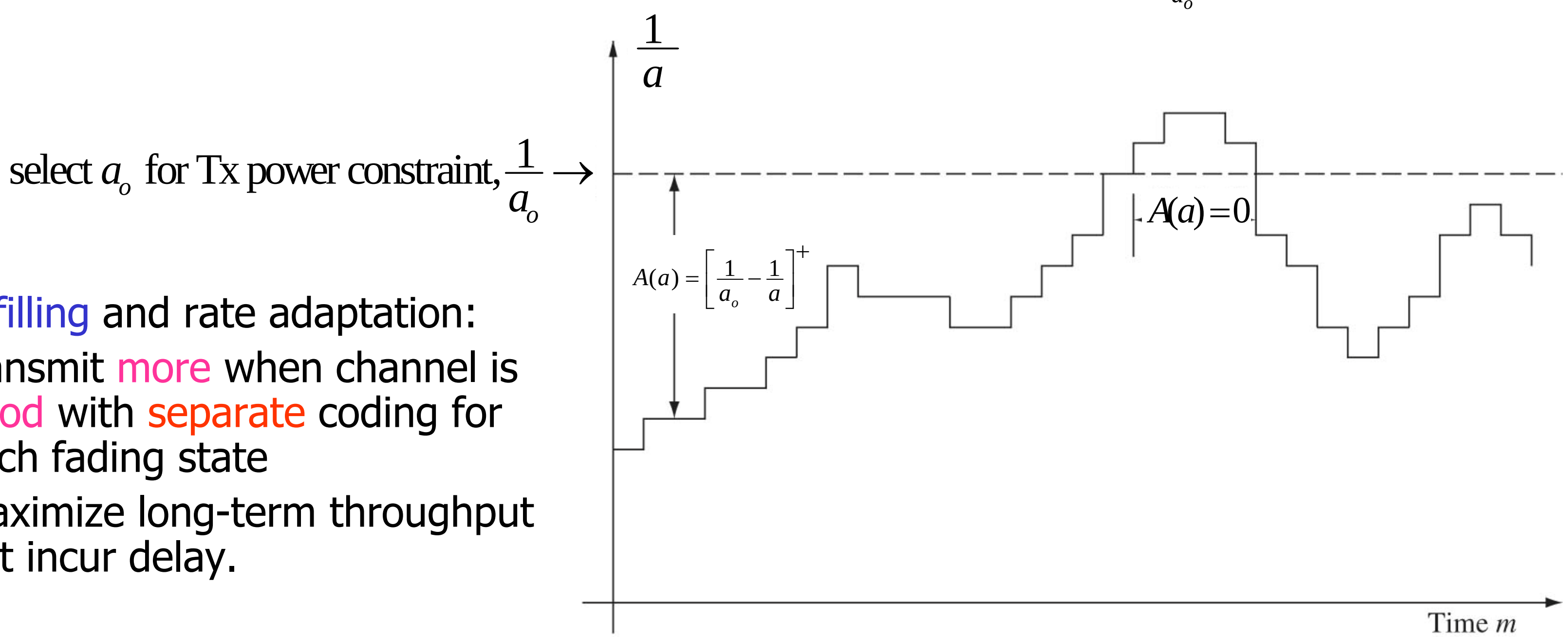
  - Greatly reduces capacity:Capacity is zero in Rayleigh fading since $E\{a^{-1}\} \rightarrow \infty$
  - achieves a delay-limited capacity.
- **Truncated inversion:** $A(a)=1/a$ only if a is above cutoff fade depth
  - to maintain constant SNR (and hence fixed rate) above cutoff
  - to increase capacity with appropriate choice of cutoff: Close to optimal

Flat- Fading Also Known at Transmitter: using Optimal Adaptive Tx power

instantaneous: $C\left[A(a)\right]=\log_2\left[1+aA(a)SNR\right]$ with Tx power constraint: $\int_0^\infty A(a)\,p(a)da=1$

$$\Rightarrow \text{ergodic:} C_A=\int_0^\infty \log_2\left[1+aA(a)SNR\right]p(a)da \Rightarrow C_{A,\max}=\max_{A(a):E\{A(a)\}=1} B\int_0^\infty \log_2\left[1+aA(a)SNR\right]p(a)da$$

solution: $A(a)=\left[\frac{1}{a_o}-\frac{1}{a}\right]^+$ select a_o for Tx power constraint $\Rightarrow C_A=B\int_{a_o}^\infty \log_2\left(aSNR/a_o\right)p(a)da.$



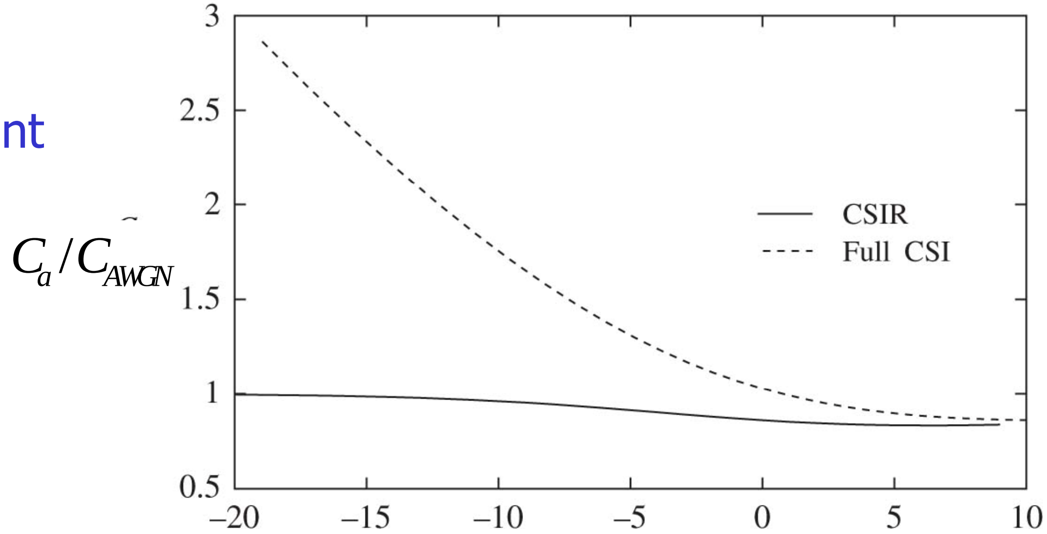
Waterfilling and rate adaptation:

- transmit more when channel is good with separate coding for each fading state
- maximize long-term throughput but incur delay.

Waterfilling: Performance

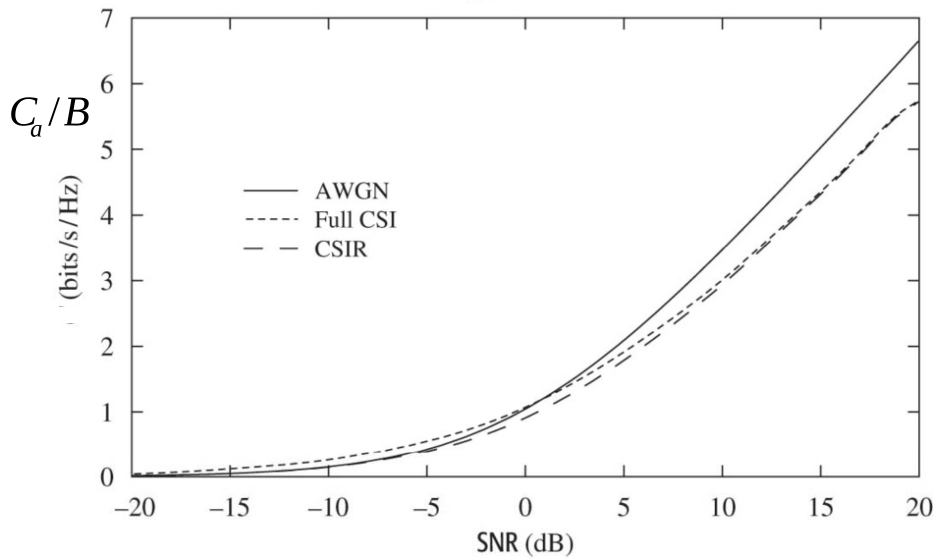
at low SNR

- Waterfilling provides a significant power gain.



at high SNR,

- waterfilling does not provide any gain, but
- CSI allows rate adaptation and simplifies coding.



Adaptive Modulation

- Adapt modulation to fading
- Parameters: Constellation size, Tx power, AMC
- Optimization criterion: Max throughput, minimum BER, or minimum Tx power.
- Example: Rate and Power Optimization to max rate for a target BER

coherent M-QAM in AWGN for $M \geq 4, 0 \leq SNR \leq 30dB$: $P_b \leq 0.2\exp\left[-1.5SNR/(M-1)\right]$,

$\rightarrow$ in presence of fading a, for a target P_b, select $A(a)$

$$\max_{A(a):E\{A(a)\}=1} E\{\log_2[M(a)]\} = \max_{A(a):E\{A(a)\}=1} E\left\{\log_2\left[1 - \frac{1.5SNR}{\ln(5P_b)} aA(a)\right]\right\}$$

$$A(a) = \begin{cases} \frac{1}{a_o} + \frac{\ln(5P_b)}{1.5a} & a \geq -\frac{a_0 \ln(5P_b)}{1.5} \\ 0 & \text{else} \end{cases}$$

$$C_{opt} = B \int_{-a_o \ln(5P_b)/1.5}^{\infty} \log_2\left(\frac{-1.5a}{a_o \ln(5P_b)}\right) p(a)da$$