# Fading Channels: Modeling

## ECSE413B - Communications Systems II - Winter 2008

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Department of Electrical & Computer Engineering
McGill University

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# Objective

1. To understand key physical parameters:
   - *Delay Spread* and *Coherence Bandwidth*
   - *Doppler Spread* and *Coherence Time*

2. To understand different types of fading:
   - *Frequency flat* (or non-selective) fading or *frequency selective* fading
   - *Slow* (or time flat) fading or *fast* (or time selective) fading

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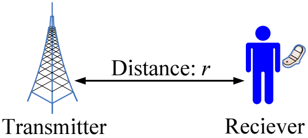
1. Free Space Model: Fixed Antenna
2. Free Space Model: Moving Antenna
3. Two-Path Model: Fixed Antenna
4. Two-Path Model: Moving Antenna
5. Wireless Fading Channels

# Free Space Model: Fixed Antenna




- Input signal at transmitter: $x(t) = \cos 2\pi f t$. What is received signal at receiver (in the absence of Gaussian noise)?

$$y(t) = \frac{\alpha \cos 2\pi f(t - r/c)}{r}, \quad c : \text{Speed of light}$$

  - $\tau = r/c$: time to travel from transmitter to receiver $\rightarrow$ Delay
  - $\alpha$: antenna patterns, e.g., antenna gains, assumed constant
  - $r$ in the denominator: Due to the decrease of electric field
- Rewrite the received signal: $\boxed{y(t) = ax(t - \tau)}$
  - $a = \alpha/r$: attenuation of the signal
  - $\tau = r/c$: delay of the signal

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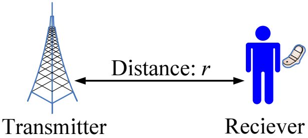


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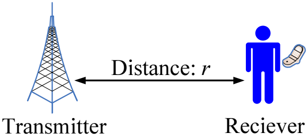


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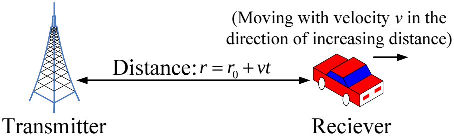


- Distance $r(t) = r_0 + vt$ depends on $t$. The received signal:

$$y(t) = \frac{\alpha \cos 2\pi f(t - r(t)/c)}{r(t)} = a(t)x(t - \tau(t))$$

Remark: Attenuation $a(t) = \frac{\alpha}{r(t)}$ and delay $\tau(t) = \frac{r(t)}{c}$ depend on $t$

- Substitute $r(t) = r_0 + vt$,

$$y(t) = a(t) \cos 2\pi f \left( (1 - \frac{v}{c})t - r_0/c \right)$$

  - The frequency is shifted an amount of $-fv/c$: from $f$ to $f - fv/c$
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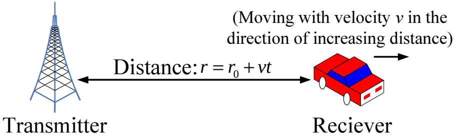


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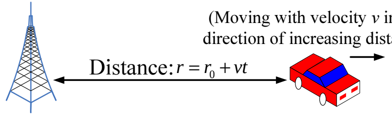


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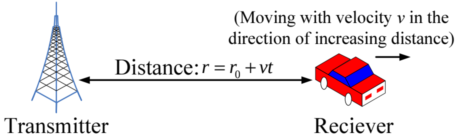


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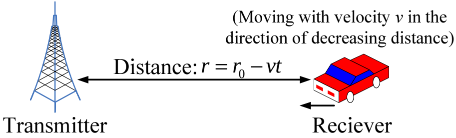


- What happens if receiver moves toward transmitter?
  - The frequency is shifted an amount of $+fv/c$: from $f$ to $f + fv/c$, e.g., higher frequency
  - Doppler shift is now $f_D = +fv/c$
- Doppler shift: Example with siren from an ambulance
  - When it moves toward us, we hear higher frequency siren
  - When it passes us, we hear a rapid shift to a lower frequency

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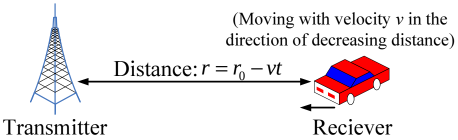


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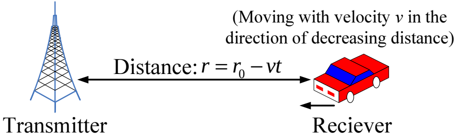


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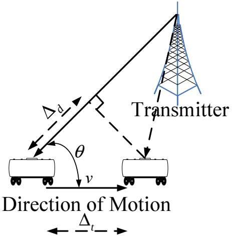


- In the previous cases, we consider that the arrival of angle of the received signal relative to the direction of motion is 0 or $\pi$. What happens if this angle is $\theta$, as shown above?
- In $\Delta_t$, receiver can travel $v\Delta_t$ in the direction of motion. That makes a change in distance $\Delta_d = v\Delta_t \cos(\theta)$ in the direction of received signal. Therefore, the Doppler shift is:

$$f_D = \pm \frac{fv}{c} \cos(\theta)$$

- Receiver moving toward transmitter: $f_D$ is positive, otherwise it is negative. The above example: positive when $-\pi/2 \leq \theta \leq \pi/2$

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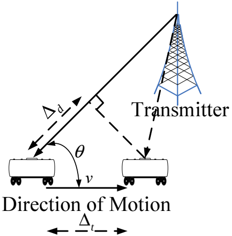


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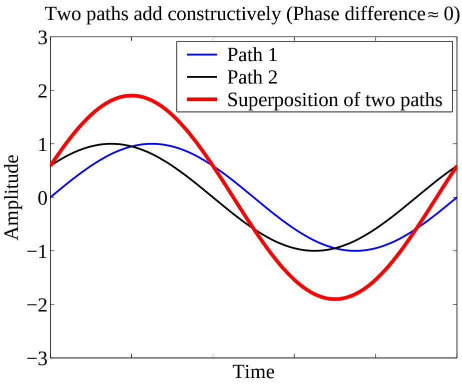
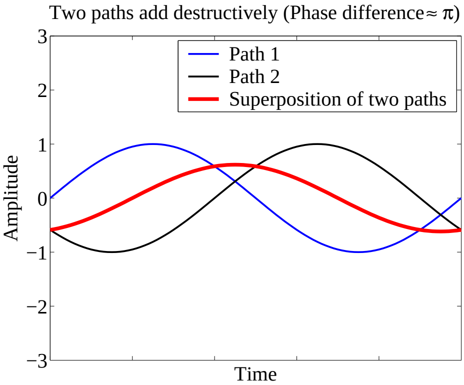
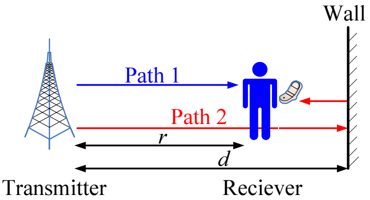
# Constructive and Destructive Interference




- Two sinusoid signals with phase difference is an integer multiple of $2\pi$: They add constructively
- Two sinusoid signals with phase difference is an odd integer multiple of $\pi$: They add destructively

# Two-Path Model: Fixed Antenna




- With $x(t) = \cos 2\pi ft$, received signal is superposition of two paths:

$$y(t) = \frac{\alpha \cos 2\pi f(t - r/c)}{r} - \frac{\alpha \cos 2\pi f(t - (2d - r)/c)}{2d - r}$$

- $y(t)$ can be re-written as $\boxed{y(t) = a_1 x(t - \tau_1) + a_2 x(t - \tau_2)}$
  $a_1 = \alpha/r; \tau_1 = r/c$: attenuation and delay of Path 1
  $a_2 = \frac{\alpha}{(2d-r)}; \tau_2 = (2d - r)/c - \frac{\pi}{2\pi f}$: attenuation and delay of Path 2
- Paths add constructively or destructively, due to phase difference:

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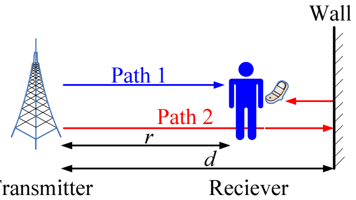


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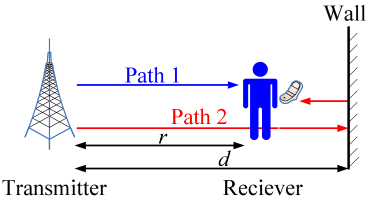


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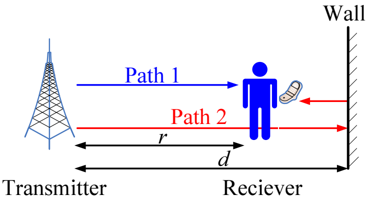


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- Consider $\Delta\theta$ as a function of $r$:
  - If at $r = r_1$, two paths add constructively, i.e., $y(t)$ is at peak, then at $r = r_1 \pm \frac{c}{4f}$, two paths add destructively, i.e., $y(t)$ is at valley? Why? You can look at the phase difference for two cases.
  - $\frac{c}{4f} = \frac{\lambda}{4}$ is called Coherence Distance: Distance from peak to valley
- Now consider $\Delta\theta = 2\pi f(\tau_2 - \tau_1)$ as a function of $f$:
  - $\boxed{T_d = |\tau_2 - \tau_1|}$ -*Delay Spread*: Difference between delays along two paths
  - Assume at $f = f_1$, two path adds constructively. It can be verified that at $f = f_1 \pm \frac{1}{2T_d}$, two path adds destructively
  - Therefore,

    *The signal $y(t)$ does not change significantly if frequency $f$ changes by an amount much smaller than* $\boxed{B_h = \frac{1}{2T_d}}$: *Coherence Bandwidth*

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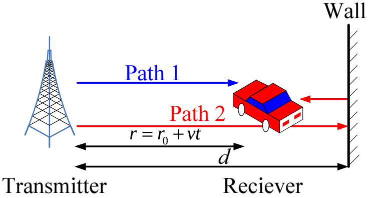


- With $x(t) = \cos 2\pi ft$, received signal is superposition of two paths

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- Due to motion:
  - Attenuations and delays depend $t$
  - Path 1: Doppler Shift of $-fv/c$; Path 2: Doppler Shift of $+fv/c \rightarrow$ Frequency can be shifted by an amount of $D_s = 2fv/c$ from $f - fv/c$ to $f + fv/c$: *Doppler Spread*
  - Coherence Distance $\frac{c}{4f} = \frac{\lambda}{4}$: Distance from Peak to Valley. With velocity $v$, *how long does this take to travel from Peak to Valley*? It is *Coherence Time* $T_c = \frac{c}{4fv}$

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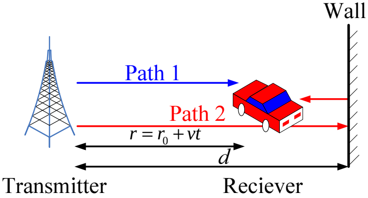


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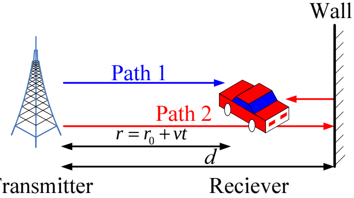


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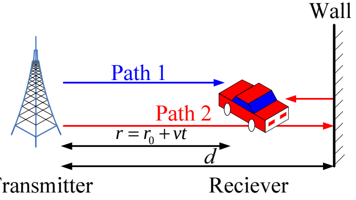


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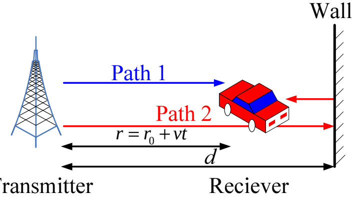
  - Attenuations and delays depend $t$
  - Path 1: Doppler Shift of $-fv/c$; Path 2: Doppler Shift of $+fv/c \rightarrow$ Frequency can be shifted by an amount of $D_s = 2fv/c$ from $f - fv/c$ to $f + fv/c$: *Doppler Spread*
  - Coherence Distance $\frac{c}{4f} = \frac{\lambda}{4}$: Distance from Peak to Valley. With velocity $v$, *how long does this take to travel from Peak to Valley*? It is *Coherence Time* $T_c = \frac{c}{4fv}$

# Two-Path Model: Moving Antenna




- With $x(t) = \cos 2\pi ft$, received signal is superposition of two paths
$$y(t) = a_1(t)x(t - \tau_1(t)) + a_2(t)x(t - \tau_2(t))$$
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# Two-Path Model: Moving Antenna (Cont.)

- *Coherence Time* $T_c$: Time to travel from Peak to Valley$\rightarrow$ In a given amount of time much smaller than $T_c$, signal does not change significantly
- *Coherence Time* $T_c = \frac{c}{4fv}$ and *Doppler Spread* $D_s = 2fv/c$. We then can see that $\boxed{D_s = \frac{1}{2T_c}}$. Does it have some meanings?
- To see the relation, let assume lengths of two paths be almost the same. Then the signal $y(t)$ is given as:

$$
\begin{aligned}
y(t) &= a_1(t)x(t-\tau_1(t)) + a_2(t)x(t-\tau_2(t)) \\
&\approx \frac{2\alpha \sin 2\pi f\left[vt/c + (r_0-d)/c\right] \sin 2\pi f\left[t - d/c\right]}{r_0 + vt}
\end{aligned}
$$

It is the product of two sinusoids, whose frequencies are of the order GHz ($f$) and Hz ($fv/c = D_s/2$).

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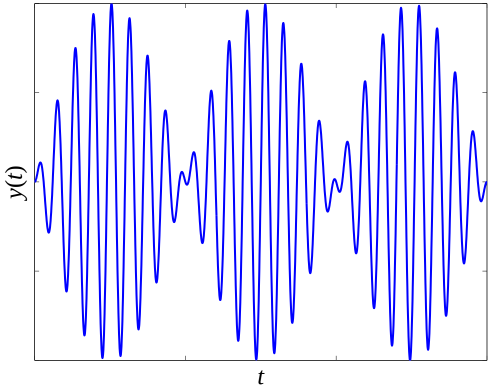


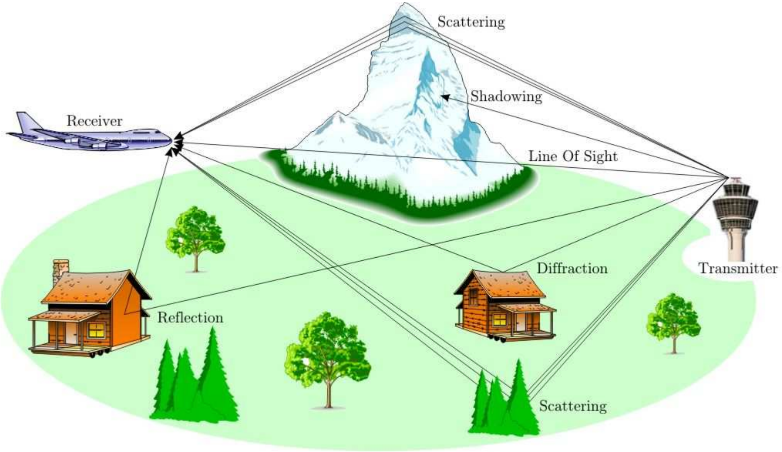

- $y(t)$ oscillating at frequency $f$ with a slowly varying envelope at frequency $D_s/2$
- From Peak of envelope to Valley of envelope: *Coherence Time*
- The *Doppler Spread* is the rate of traversal across the Peaks and Valley → Inversely proportional to the *Coherence Time*

# Two-Path Model: Summary

- The significant change of the received signal $y(t)$ is due to the phase changes.
- The received signal $y(t)$ can changes significantly when $f$ changes by $B_h = \frac{1}{2T_d}$, where $T_d$, *Delay Spread*, is the difference between the delays of two paths. $B_h$ is called *Coherence Bandwidth*
- Due to the motion:
  - Each path has its own Doppler Shift $D_i$, $i = 1, 2$ and one has the *Doppler Spread* $D_s = |D_1 - D_2|$.
  - The received signal $y(t)$ can also changes significantly when $t$ changes $T_c = \frac{1}{2D_S}$, which is called *Coherence Time*
- The arguments here can be extended to the case of *multi-path fading channels*, where we have many (physical) paths

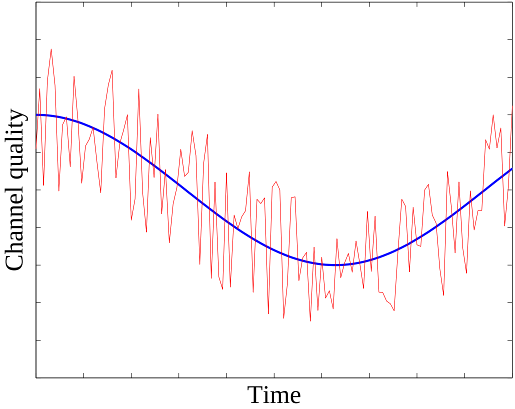
# Outline

# Wireless Communications




Multiple received versions of the same transmitted signal caused by reflections → *multipath*

# Wireless Fading Channels




- With many paths, they might add destructively or constructively: channel strengths change *randomly* with time → Fading
- When channel is weak, i.e., bad quality→Low reliability

# Input/Output Model

- With input $x(t)$, output $y(t)$ is:

$$y(t) = \sum_i a_i(t)x(t - \tau_i(t))$$

where $a_i(t)$ and $\tau_i(t)$ are attenuation and delay of path $i$th. Note: path $i$th refers to a physical path and there are so many of them

- The input/output relationship is now written as:

$$y(t) = \int_{-\infty}^{+\infty} h(\tau, t)x(t - \tau)\mathrm{d}\tau$$

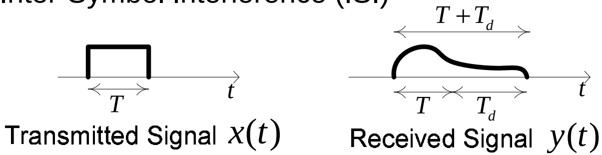
with impulse response $h(\tau, t) = \sum_i a_i(t)\delta(\tau - \tau_i(t))$: the response $h(t, \tau)$ at time $t$ to an impulse transmitted at time $t - \tau$

- In general, $a_i(t)$ & $\tau_i(t)$ depend on $t \rightarrow$ (Linear) time-variant channel.
- The time-varying frequency response:

$$H(f; t) = \int_{-\infty}^{+\infty} h(\tau, t)\exp(-j2\pi f\tau)\mathrm{d}\tau = \sum_i a_i(t)\exp(-j2\pi f\tau_i(t))$$

# Key Physical Parameters

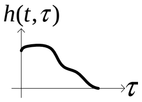
- Different $\{\tau_i(t)\} \rightarrow x(t)$ is dispersive in time, which causes a so-called Inter-Symbol Interference (ISI)



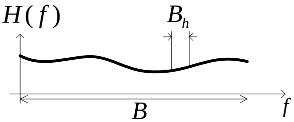

- *Delay Spread*: $\boxed{T_d = \max_{i,j} |\tau_i(t) - \tau_j(t)|}$: Time delay between the arrival of the first received signal component and the last received signal component.
- By comparing $T_d$ with symbol duration $T$, we have:
  - $T_d \ll T$, ISI is negligible→*frequency non-selective* (or flat) fading
  - $T_d \gg T$, ISI is severe→*frequency selective* fading
  - But why we call it *frequency selective*?

# Key Physical Parameters (Cont.)

- Basically, dispersion in time means selectivity in domain
- We know that the channel changes significantly is due to the phase change, which can be represented by $2\pi f T_d$. Given $T_d$,
  - When $f$ changes an amount of $B_h = \frac{1}{2T_d}$, $2\pi f T_d = \pi$ and channel changes significantly. $B_h$ is called *Coherence Bandwidth* of the channel
  - Equivalently, if $f$ changes an amount that is much smaller than $B_h$, we can consider channel stays the same
- Now, with symbol duration is $T$, the bandwidth occupied is $B \approx \frac{1}{T}$
  - If $T_d \gg T$, bandwidth $B = 1/T \gg B_h \rightarrow H(f)$ changes differently over band $B \rightarrow$ *frequency selective*




Channel Impulse Reponse




Frequency Response

- ISI in $t$ domain is equivalent to frequency selectivity in $f$ domain

# Key Physical Parameters (Cont.)

- From the case of two-path, we know that channel not only changes over $f$ but also change over $t$: *Coherence Time* and *Doppler Spread*
- Motion with velocity $v(t)$:
  - Each path $i$ has velocity $v_i(t)$ with which the $i$th path length is increasing (or decreasing). The relation between $v(t)$ and $v_i(t)$ depends on the arrival of angle of the received signal on path $i$
  - Let $\tau_i^{'}(t) = v_i(t)/c$. Doppler shift for each path $i$: $-f\tau_i^{'}(t)$ (the path length is increasing) or $+f\tau_i^{'}(t)$ (the path length is decreasing)
  - With center frequency $f_c$, *Doppler Spread* is the largest difference between Doppler shifts (counting paths that are not too weak):

$$D_s = \max_{i,j} f_c |\tau_i^{'}(t) - \tau_j^{'}(t)|$$

  - Similar to two-path model, we have the *Coherence Time*:

$$T_c = \frac{1}{2D_s}$$

  - Note: one can replace the factor of 2 by 4, or even 8, depending on the view of phase change: Some consider $\pi$ is significant, others might user $\pi/2$, $\pi/4$

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- Recall in two-path model, *Coherence Time* $T_c$ is defined as time to travel from Peak to Valley. Similarly, in multi-path model, $T_c$ is defined as the interval over which the channel changes significantly as a function of $t$
- *Coherence Time* $T_c$ & *Doppler Spread* $D_s$ show how quickly channel changes over time, hence
  - Channels are often categorized as *fast fading* and *slow fading*
  - In general, if $T_c \gg \frac{1}{B}$, i.e., channel keeps constant over many symbol durations, we have *slow (or time flat) fading*. Otherwise, it is *fast (or time selective) fading*
  - Clearly, fast or slow not only depends on the environment but also on the application
- One also can compare *Coherence Time* $T_c$ and *Delay Spread* $T_d$. Normally, $T_d$ is of the order of microseconds, while it is of the order of microseconds for $T_s$: $T_d \ll T_c$ and it is called *underspread* channel

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# Summary of Types of Wireless Channels and Defining Characteristic

| Types of channel | Defining characteristic |
|---|---|
| Flat fading | $B \ll B_h$ |
| Frequency selective fading | $B \gg B_h$ |
| Fast fading | $T_c \ll \frac{1}{B}$ |
| Slow fading | $T_c \gg \frac{1}{B}$ |
| Underspread | $T_d \ll T_c$ |

- $B$: Communication Bandwidth
- $T_d$ and $B_h$: *Delay Spread* and *Coherence Bandwidth*
- $T_c$ and $D_s$: *Coherence Time* and *Doppler Spread*

# An Example: Representative Values of The Physical Parameters

| Key Channel Parameters | Symbol | Values |
|---|---|---|
| Carrier Frequency | $f_c$ | 1 Ghz |
| Communication Bandwidth | $B$ | 1 MHz |
| Distance | $d$ | 1 km |
| Velocity of Mobile | $v$ | 60 km/h |
| Doppler Shift | $D = f_c v/c$ | 50 Hz |
| Doppler Spread | $D_s$ | 100 Hz |
| Coherence Time | $T_c = \frac{1}{2D_s}$ | 5 ms |
| Delay Spread | $T_d$ | 1 $\mu$s |
| Coherence Bandwidth | $B_h = \frac{1}{2T_d}$ | 500 kHz |

# What Next?

- Taking into account Gaussian noise, we will study the complex baseband representation of the model

$$y(t) = \sum_i a_i(t)x(t - \tau_i(t)) + w(t)$$

- Using sampling theorem, we will consider the discrete-time baseband model
  - Single-tap model, which corresponds to frequency flat fading
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- We will end up with statistical channels models, which are very helpful for the system analysis and design

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